EDAMAME Solutions

NYCMT

October 2025

Problem 1. The polynomial x^2+5x+1 has roots a and b. Evaluate $\left(a+\frac{1}{a}\right)\left(b+\frac{1}{b}\right)$.

Answer. $\boxed{25}$

Solution. Notice that by Vieta's Formulas, $a+b=-5$ and $ab=1$. Since $ab=1$, we know that $\frac{1}{a}=b$ and $\frac{1}{b}=a$, so our desired value is

$$(a+b)(b+a)=(-5)(-5)=\boxed{25}.$$

□

Problem 2. Compute the least positive integer n such that n^2 has an even number of digits and has 999 as its leftmost three digits.

Answer. $\boxed{9995}$

Solution. We rewrite n^2 as $(10^k - a)^2$. Since it has 999 as its leftmost digits, n^2 should be close to a power of 10, and because it has an even number of digits, n^2 is close to an even power of 10. We then expand:

$$(10^k - a)^2 = 10^{2k} - 2a \cdot 10^k + a^2.$$

If $k \leq 3$, note that the leftmost three digits cannot be 999, since $9^2 = 81$, $99^2 = 9981$, and $999^2 = 998001$. However, since $9999^2 = 99980001$ satisfies the condition, the least possible n should be close to 9999. We write:

$$(10^4 - a)^2 = 10^8 - 2a \cdot 10^4 + a^2.$$

We want $(10000 - a)^2 \geq 999 \cdot 10^5$, which translates to the condition

$$2a \cdot 10^4 - a^2 \leq 10^5.$$

Dividing by 10^4 on both sides, we get that

$$a - \frac{a^2}{20000} \leq 5.$$

Thus, the largest possible value of a is 5, so the smallest possible value of n is $10^4 - 5 = \boxed{9995}$. □

Problem 3. Takaki writes down a two digit number (possibly with a leading zero) to send to Akari. Then, James, Kyle, and Sophia take turns randomly choosing one of the digits and changing it to a randomly chosen different digit. When Sophia finishes her turn, she sends the number to Akari. What is the probability that Akari gets the original number?

Answer. $\boxed{\frac{2}{81}}$

Solution. First, James can change any digit to a different digit without affecting the outcome. However, if Kyle changes the two digit number back to the original number, it will be impossible for the written number to be the same as the original after Sophia changes one of the digits. Similarly, if Kyle changes a different digit from the one James changed, both digits will be different from the original. Thus, it will be impossible for Sophia to change the number back to the original. Due to this, there is an 8/18 probability that Kyle changes a digit so that it is still possible for the number to be changed back to the original—he has to change the digit James changed and not change it to the digit it originally was.

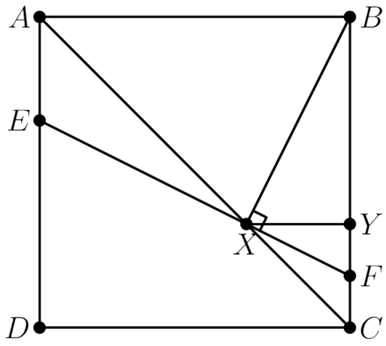
After that, Sophia has a 1/18 probability of choosing the digit James and Kyle changed and changing it to the digit it originally was. Thus, the probability Akari gets the original number is

$$\frac{8}{18} \times \frac{1}{18} = \boxed{\frac{2}{81}}.$$

□

Problem 4. Let $ABCD$ be a unit square with point F on BC and point E on AD, and let X denote the intersection of EF and AC. If $AE = 2CF$ and $BX \perp EF$, compute the length of segment CF.

Answer. $\boxed{1/6}$



Solution. We place the diagram on the coordinate plane, with $A = (0, 1)$, $B = (1, 1)$, $C = (1, 0)$, and $D = (0, 0)$. Notice that because $\angle XAE = \angle XCF = 45^\circ$ and $\angle AXE = \angle CXF$, we have that

$$\triangle AXE \sim \triangle CXF$$

by AA similarity. Because $AE = 2CF$ and the triangle are similar, we also know that the distance from X to line AE is the same as the distance from X to line CF. Thus, the x-coordinate of X is $2/3$. Since we also know that X lies on AC, which is defined by $y = 1 - x$, we now know that $X = (2/3, 1/3)$.

Let $Y = (1, 1/3)$ be the projection of X onto BC. We have that $BY = 2/3$ and $XY = 1/3$. Because $\triangle BXF$ is a right triangle, we can see that $\triangle BXY \sim \triangle XFY$. Thus,

$$\frac{FY}{XY} = \frac{XY}{BY}$$

and from here, we obtain that $FY = \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6}$. Finally, we find that

$$CF = 1 - BY - YF = 1 - \frac{2}{3} - \frac{1}{6} = \boxed{\frac{1}{6}},$$

as desired. □

Problem 5. Let k be a real number such that the polynomial $p(x) = x^3 - 6x^2 + 36x - k$ has three distinct roots z_1, z_2, and z_3 which form a nondegenerate right triangle when plotted in the complex plane. Find the sum of all possible values of k.

Answer. $\boxed{112}$

Solution. First, notice that the roots cannot all be real. Otherwise, they will all be on the real line in the complex plane and will form a degenerate triangle. Thus, the polynomial will have two complex nonreal roots. WLOG let them be z_1 and z_2. Since they must be conjugates of each other, the perpendicular bisector of the line between them is the real axis, so z_3 is equidistant from z_1 and z_2. From this, we conclude that the triangle is a 45-45-90 right triangle, with a right angle between the lines formed by z_1 and z_3, and z_2 and z_3. Thus, the roots must be of the form $a + bi$, $a - bi$, $a + b$.

By Vieta's Formulas, we find that

$$(a + bi) + (a - bi) + a + b = 6 \text{ and}$$
$$(a + bi)(a - bi) + (a - bi)(a + b) + (a + b)(a + bi) = 36.$$

Simplifying, we obtain the equations

$$\begin{cases} 3a + b = 6 \\ 3a^2 + b^2 + 2ab = 36. \end{cases}$$

We rearrange the first equation to get that $b = 6 - 3a$, and substitute it into the second equation to obtain the following:

$$3a^2 + (6 - 3a)^2 + 2a(6 - 3a) = 36,$$

which eventually simplifies to

$$6a^2 - 24a = 0.$$

Thus, we find that (a, b) is either $(0, 6)$ or $(4, -6)$. If $(a, b) = (0, 6)$ then

$$k = z_1z_2z_3 = (0 + 6i)(0 - 6i)(0 + 6) = 216,$$

and if $(a, b) = (4, -6)$ we find that

$$k = z_1z_2z_3 = (4 - 6i)(4 + 6i)(4 - 6) = -104.$$

Thus, the sum of the possible values of k are $216 - 104 = \boxed{112}$. □

Problem 6. Let a, b be positive integers with $a > b \geq 3$. A regular a-gon and b-gon sharing a vertex are inscribed in the same circle and their vertices and the intersections between their sides are marked. If the total number of marked points is 35, find the sum of all possible values of a.

Answer. $\boxed{115}$

Solution. Let $d = \gcd(a, b)$. Consider the shared vertices on the circle, which is equal to d. Then, between every two such vertices (exclusive), there are $a/d - 1$ a-gon vertices and $b/d - 1$ b-gon vertices. Counting the number of vertices inside the circle on each segment, we see that the sides of the b-gon cross the sides of the a-gon twice for each vertex since there is at least one a-gon vertex between every two b-gon vertices. This gives $2(b/d - 1)$ interior intersections for each of these d segments. Thus, we find that

$$35 = d(a/d - 1 + b/d - 1 + 2(b/d - 1) + 1) = a + 3b - 3d.$$

We may now do casework on the value of d, noting that both sides are divisible by d so $d \mid 35$:

- $d = 1$. Then $a + 3b = 38$. From $a > b \geq 3$, we find that $(a, b) = (35, 1), (32, 2), \ldots, (11, 9)$. From $\gcd(a, b) = 1$, we may reduce to the 4 cases $(29, 3), (23, 5), (17, 7), (11, 9)$.
- $d \neq 1$. Then we find that $d = 5, 7, 35$ so $a/d + 3(b/d) = (35/d) + 3 = 10, 8, 6$ for integers $a/d > b/d$, and in every case we must have either $b/d = 1$ or $a/d = 4$, $b/d = 2$. Since $\gcd(a/d, b/d) = 1$, we only have $b/d = 1$ so $b = d$ and
$$35 = a + 3b - 3d = a \implies a = 35.$$

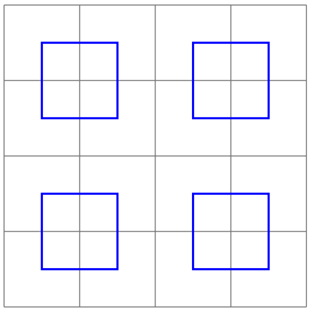
Adding up these cases gives $29 + 23 + 17 + 11 + 35 = \boxed{115}$. $\square$

Problem 7. One person is standing in each cell of a 4×4 grid. How many ways are there for each person to move to an orthogonally adjacent cell such that each cell is occupied afterwards and no pair of people swapped places?

Answer. $\boxed{88}$

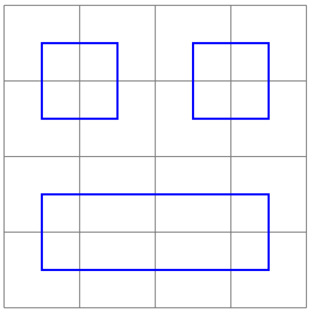
Solution. Drawing a directed edge between two orthogonally adjacent squares if a person went from one square to the other, we see that every vertex (cell) has indegree and outdegree exactly 1. That is, every configuration of moving people corresponds to a decomposition of the grid into directed cycles of length greater than 2. We now consider casework on the sizes of the cycles.

If there are 4 cycles of length 4, we see that there is only one working configuration for the undirected cycles:



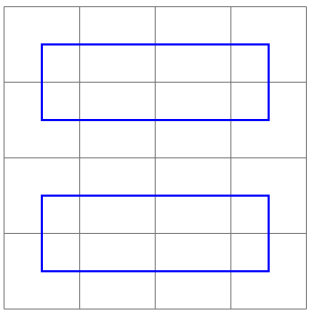
This gives $2^4 = 16$ ways by choosing the direction in each cycle.

If there are 2 cycles of length 4 and one of length 8, there are 4 working configurations for the undirected cycles, given by the rotations of the following configuration:



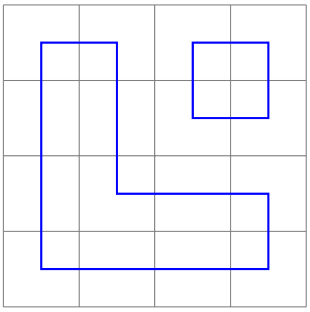
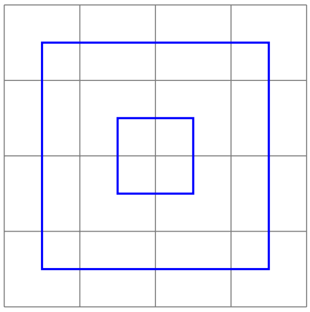
This gives $4 \cdot 2^3 = 32$ ways by choosing the direction in each cycle.

If there are 2 cycles of length 8, there are 2 working configurations for the undirected cycles, given by the rotations of the following configuration:



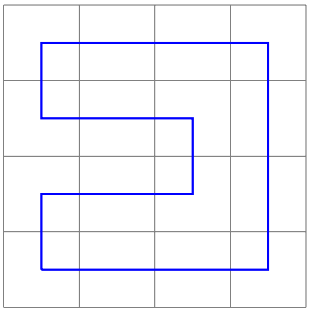
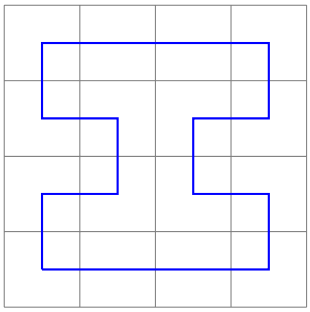
This gives $2 \cdot 2^2 = 8$ ways by choosing the direction in each cycle.

If there is one cycle of length 4 and one cycle of length 12, there are 5 working configurations for the undirected cycles, given by the rotations of the following configurations:

This gives $5 \cdot 2^2 = 20$ ways by choosing the direction in each cycle.

Finally, if there is one cycle of length 16, there are 6 working configurations for the undirected cycles, given by the rotations of the following configurations:

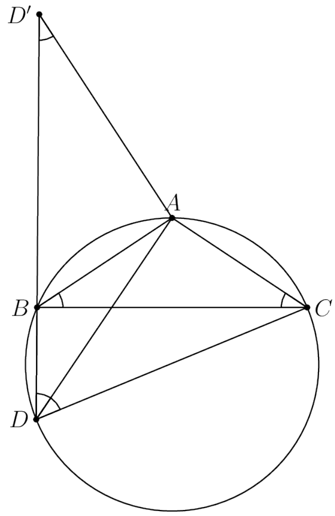
This gives $6 \cdot 2 = 12$ ways by choosing the direction in each cycle.

Now, observe that no cycle can have odd length by considering the number of color switches on a checkerboard coloring of the grid. Furthermore, no cycle can have length 6, since there is only one shape for a cycle of length 6 (a 3×2 rectangle) and no matter where it is positioned the remaining parts cannot be partitioned into cycles. Similarly, no cycle can have length 10 since that would force a cycle of length 6. We conclude these are the only possibilities, so the answer is $16 + 32 + 8 + 20 + 12 = \boxed{88}$.

$\square$

Problem 8. Triangle $\triangle ABC$ with $AB = AC = 6$ and $BC = 10$ is inscribed in circle Ω. A point D is on Ω with $AD = 9$. Find the area of quadrilateral $ABDC$.

Answer. $\boxed{\dfrac{45\sqrt{11}}{4}}$



Solution 1. Rotate $\triangle ACD$ about A so that C is mapped to B and D is mapped to a point D'. Since $\angle ACD + \angle DBA = 180°$ by the cyclicity of $ABDC$, it follows that $\angle ABD' + \angle DBA = 180°$ so D', B, and D are collinear. Thus quadrilateral $ABDC$ has the same area as $\triangle AD'D$. We see that $AD = AD' = 9$ and

$$DD' = BD' + BD = BD + CD$$

where $6BD + 6CD = AD \cdot BC$ by Ptolemy's theorem, so $BD + CD = 15$ and $AD'D$ is a $9-9-15$ triangle. We may calculate the length of the altitude from A to $D'D$ as $\sqrt{9^2 - (15/2)^2} = \frac{3\sqrt{11}}{2}$ so that the area is

$$\frac{1}{2} \cdot 15 \cdot \frac{3\sqrt{11}}{2} = \boxed{\frac{45\sqrt{11}}{4}},$$

as desired. □

Solution 2. We wish to find $[\triangle ABD] + [\triangle ACD]$. By Sine Area, this is equal to

$$\frac{1}{2} \cdot AB \cdot AD \cdot \sin\angle BAD + \frac{1}{2} \cdot AC \cdot AD \cdot \sin\angle CAD = 27 \cdot (\sin\angle BAD + \sin\angle CAD).$$

Let R be the circumradius of Ω. By the Extended Law of Sines,

$$\sin\angle BAD + \sin\angle CAD = \frac{BD}{2R} + \frac{CD}{2R}.$$

As in the above solution, we find that $BD + CD = 15$. To find R, we use Heron's Formula and the identity $[\triangle ABC] = \frac{abc}{4R}$ to find that

$$R = \frac{1}{4} \cdot \frac{6 \cdot 6 \cdot 10}{\sqrt{11 \cdot 5 \cdot 5 \cdot 1}} = \frac{18\sqrt{11}}{11}.$$

Thus the answer is

$$27 \cdot \frac{15}{2 \cdot 18\sqrt{11}/11} = \boxed{\frac{45\sqrt{11}}{4}}.$$

□

Problem 9. Given that p is a prime number and

$$\frac{1}{p} = 0.\overline{0002a240609997175b39}$$

for some unknown digits a and b, compute p.

Answer. $\boxed{3541}$

Solution. First, notice that since $0.0002 < \frac{1}{p} < 0.0003$, we know that $3333 < p < 5000$. We can rewrite the given expression as

$$\frac{1}{p} = \sum_{k=1}^{\infty} \frac{\overline{2a240609997175b39}}{10^{20k}} = \frac{\overline{2a240609997175b39}}{10^{20}-1}.$$

Thus, we find that

$$p = \frac{10^{20}-1}{\overline{2a240609997175b39}}.$$

Notice that $10^2 - 1 \mid 10^{20} - 1$. However, because of our bounds for p, we know that p is not equal to 9 or 11, which means that $\overline{2a240609997175b39}$ must be divisible by 99. Using our divisibility rules for 9 and 11, we obtain the following modular congruences:

$$\begin{cases} a + b + 1 \equiv 0 \pmod{9} \\ a + 4 \equiv b + 3 \pmod{11}. \end{cases}$$

Since $0 \le a, b \le 9$, we find that the only solution is $(a, b) = (8, 9)$. Thus, we have that

$$\frac{1}{p} = 0.\overline{00028240609997175939}$$

Since, we know the division on the right hand side will give an integer, we can divide 10000 by 2.824 to the nearest integer to get 3541. This division gives $p = \boxed{3541}$. $\square$

Problem 10. Complex numbers a, b, and c satisfy the equations

$$\begin{aligned} ab + b + c + 1 &= 0 \\ bc + c + a + 1 &= 0 \\ ca + a + b + 1 &= 0. \end{aligned}$$

Find the sum of all possible values of a^2.

Answer. $\boxed{7}$

Solution. Pairwise subtracting the equations, we get

$$\begin{aligned} b(a-c) &= a-b \\ c(b-a) &= b-c \\ a(c-b) &= c-a \end{aligned} \quad \iff \quad \begin{aligned} (b-1)(a-c) &= c-b \\ (c-1)(b-a) &= a-c \\ (a-1)(c-b) &= b-a. \end{aligned}$$

Now, consider whether a, b, c are all distinct. If they are not then assume without loss of generality that $a = b$. It follows that $b - c = c(b - a) = 0$ so $a = b = c$ and it follows that $a^2 + 2a + 1 = 0$ so $a = b = c = -1$. If a, b, c are all distinct then multiplying the two columns of three equations obtained above and dividing by $(a - b)(b - c)(c - a)$ gives $(a - 1)(b - 1)(c - 1) = 1$ and $abc = -1$. Subtracting these gives

$$-2 = abc - (a-1)(b-1)(c-1) = ab + bc + ca - a - b - c + 1.$$

Adding the original 3 equations gives

$$ab + bc + ca + 2(a + b + c) = -3.$$

Solving this as a system of linear equations in $ab + bc + ca$ and $a + b + c$ gives $a+b+c = 0$, $ab+bc+ca = -3$. Since $abc = -1$, we get by reverse Vieta's formulas that $a^3 - 3a + 1 = 0$. Thus the sum of all possible values of a^2 is $0^2 - 2(-3) = 6$ in this case. This gives an answer of $6 + 1 = \boxed{7}$.

$\square$

Problem 11. On the coordinate plane, Elma is at $(0,0)$ and Visby is at $(32,32)$. Each minute, Elma chooses to either move one unit to the right or one unit upwards with equal probability. She then moves in that direction if and only if her coordinate in that direction is less than 32 (for example, if she is at the point (4, 32) and chooses to go upwards, she doesn't move that minute). Compute the expected number of minutes that Elma will not move before she reaches Visby.

Answer. $\boxed{\frac{\binom{64}{32}}{2^{58}}}$

Solution. We find the number of minutes and subtract 64. It is an equivalent problem to find the expected number of minutes to enter the region $x \geq 32, y \geq 32$, given unrestricted random upward and rightward movement. We can enter this region at any point of the form $(x, 32)$, or $(32, y)$ for $x, y \geq 32$. We consider casework on which direction Elma enters from: below or left. In the case of below, for each of the points $(x, 32)$, the probability Elma enters there is the probability she reaches $(x, 31)$ at some point and then goes up, or $\frac{1}{2}\frac{\binom{x+31}{31}}{2^{x+31}}$, and a similar probability occurs for the $(32, y)$ case. Thus, we may merge them to get rid of the one-half. This gives an expression of

$$\sum_{n=32}^{\infty}(n+32)\frac{\binom{n+31}{31}}{2^{n+31}}$$

for the expected value. From the fact that $(a+1)\binom{a}{b} = (b+1)\binom{a+1}{b+1}$, we may rewrite the sum as

$$\sum_{n=32}^{\infty}\frac{\binom{n+32}{32}}{2^{n+26}} = \frac{1}{2^{26}}\sum_{n=32}^{\infty}\frac{\binom{n+32}{32}}{2^{n}}.$$

From the generating function for $(1-x)^{-33}$ we get that[1]

$$\sum_{n=0}^{\infty}\frac{\binom{n+32}{32}}{2^{n}} = \left(1-\frac{1}{2}\right)^{-33}$$

so that our sum can be rewritten as

$$\frac{2^{33}}{2^{26}} + \frac{1}{2^{26}}\frac{\binom{64}{32}}{2^{32}} - \frac{1}{2^{26}}\sum_{n=0}^{31}\frac{\binom{n+32}{32}}{2^{n}} = 128 - \frac{1}{2^{26}}\sum_{n=0}^{31}\frac{\binom{n+32}{32}}{2^{n}} + \frac{\binom{64}{32}}{2^{58}}.$$

We now show by induction the identity

$$\sum_{n=0}^{k}\frac{\binom{n+k}{k}}{2^{n}} = 2^{k}.$$

For the base case of $k=1$, this is easily checked. Now, from the identity for $k-1$, we find that

$$\sum_{n=0}^{k}\frac{\binom{n+k}{k}}{2^{n}} + 2\sum_{n=0}^{k-1}\frac{\binom{n+k-1}{k-1}}{2^{n}} = \sum_{n=0}^{k-1}\frac{\binom{n+k-1}{k-1}+\binom{n+k-1}{k}}{2^{n-1}} + \frac{\binom{2k-1}{k}}{2^{k-1}} + \frac{\binom{2k}{k}}{2^{k}} = \sum_{n=0}^{k}\frac{\binom{n+k}{k}}{2^{n-1}},$$

implying the desired result by substituting the identity for $k-1$. Plugging this into our expression and subtracting 64 gives an answer of $\boxed{\frac{\binom{64}{32}}{2^{58}}}$. □

[1]See Exercise 5.7 of Evan Chen's *Summations* handout.

Problem 12. Evaluate the sum

$$\sum_{n=0}^{\infty} \frac{\sin^4\left(\frac{2^n\pi}{101}\right)}{4^n}.$$

Answer. $\boxed{\sin^2\left(\frac{\pi}{101}\right)}$ or $\boxed{\frac{1}{2} - \frac{\cos\left(\frac{2\pi}{101}\right)}{2}}$

Solution 1. Notice that

$$\sin^4(\theta) = \sin^2(\theta)\cdot(1-\cos^2(\theta)) = \sin^2(\theta) - \sin^2(\theta)\cos^2(\theta) = \sin^2(\theta) - \frac{\sin^2(2\theta)}{4}.$$

Thus, we can rewrite the sum as

$$\sum_{n=0}^{\infty} \frac{\sin^2\left(\frac{2^n\pi}{101}\right)}{4^n} - \frac{\sin^2\left(\frac{2^{n+1}\pi}{101}\right)}{4^{n+1}} = \sum_{n=0}^{\infty} \frac{\sin^2\left(\frac{2^n\pi}{101}\right)}{4^n} - \sum_{n=1}^{\infty} \frac{\sin^2\left(\frac{2^n\pi}{101}\right)}{4^n}.$$

Thus, the sum is equal to $\boxed{\sin^2\left(\frac{\pi}{101}\right)}$. □

Solution 2. Note that $\sin(\theta) = \frac{e^{i\theta}-e^{-i\theta}}{2i}$, so

$$\sin^4(\theta) = \left(\frac{e^{i\theta}-e^{-i\theta}}{2i}\right)^4 = \frac{e^{4i\theta} - 4e^{2i\theta} + 6 - 4e^{-2i\theta} + e^{-4i\theta}}{16} = \frac{\cos(4\theta)}{8} - \frac{\cos(2\theta)}{2} + \frac{3}{8}.$$

Thus, we can rewrite the sum as

$$2\sum_{n=0}^{\infty} \frac{\cos\left(\frac{2^{n+2}\pi}{101}\right)}{4^{n+2}} - \frac{\cos\left(\frac{2^{n+1}\pi}{101}\right)}{4^{n+1}} + \frac{3}{4^{n+2}} = 2\sum_{n=2}^{\infty} \frac{\cos\left(\frac{2^n\pi}{101}\right)}{4^n} - 2\sum_{n=1}^{\infty} \frac{\cos\left(\frac{2^n\pi}{101}\right)}{4^n} + 2\sum_{n=2}^{\infty} \frac{3}{4^n}.$$

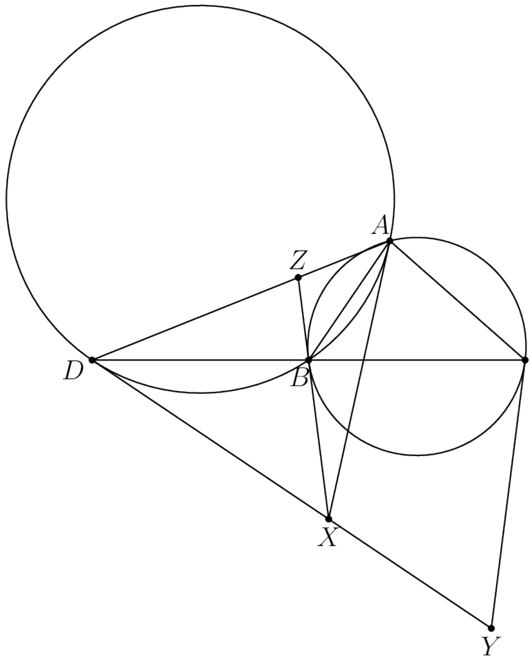
This simplifies to

$$-2\cdot\frac{\cos\left(\frac{2\pi}{101}\right)}{4} + 2\cdot\frac{1}{4} = \boxed{\frac{1}{2} - \frac{\cos\left(\frac{2\pi}{101}\right)}{2}},$$

as desired. □

Problem 13. Let $\triangle ABC$ be a triangle with $AB = 8$, $AC = 10$, and $BC = 12$. Point D is chosen on line BC such that B is between D and C. Let Ω_1 be circumcircle of $\triangle ABC$ and Ω_2 be circumcircle of $\triangle ABD$. Let tangents to Ω_2 at A and Ω_1 at B meet at X, and the tangent to Ω_2 at D and tangent to Ω_1 at C meet at Y. If D, X, Y are collinear, compute the length XY.

Answer. $\boxed{\dfrac{11\sqrt{79}}{9}}$



Solution. We begin by claiming that if D, X, and Y are collinear, then D must be the reflection of C over B. Indeed, note that the collinearity is equivalent to the assertion that line XB, which is the B-symmedian of triangle ABD because of its concurrence with the tangent intersection point, is tangent to Ω_1 at B. Then, by the Ratio Lemma, using the fact that the symmedian and median are isogonal conjugates, we get

$$\frac{AB}{BD} = \frac{\sin(\angle ZBA)}{\sin(\angle ZBD)}.$$

Let point Z be the intersection of line BX with segment AD. Note that $\angle ZBD = \angle XBC = \angle BAC$, where we make use of the angle condition for the tangency of BX. Similarly, we get $\angle ZBA = \angle ACB$. By the law of sines in triangle ABC, we get

$$\frac{BC}{\sin(\angle BAC)} = \frac{AB}{\sin(\angle ACB)} \implies \frac{AB}{BC} = \frac{\sin(\angle ACB)}{\sin(\angle BAC)} = \frac{\sin(\angle ZBA)}{\sin(\angle ZBD)} = \frac{AB}{BD}$$

giving $BC = BD$, as desired. Note that $XY = DY - DX$, and thus it suffices to compute the two lengths on the right hand side. From tangency, we get $\angle CDY = \angle DAB$ and $\angle DCY = \angle BAC$, so we have

$$\angle DYC = 180° - \angle DCY - \angle CDY = 180° - \angle BAC - \angle DAB = \angle 180° - \angle DAC$$

giving that $ACYD$ is a cyclic quadrilateral. Thus, $\angle DAB = \angle CDY = \angle CAY$ and $\angle ADB = \angle ADC = \angle AYC$, so $\triangle ABD \sim \triangle ACY$. From Stewart's Theorem on $\triangle ACD$, we get $AD = 2\sqrt{79}$. Applying our similarity with ratio $\frac{AC}{AB} = \frac{5}{4}$, we get $CY = 15$ and $AY = \frac{5\sqrt{79}}{2}$. From Ptolemy on $ABYD$, we now get $DY = 3\sqrt{79}$. Note that $\angle ADX = \angle ADB + \angle BDX = 180° - \angle ABD$. From the law of cosines on $\triangle ABD$, we get $-\cos\angle ABD = \cos\angle ADX = \frac{9}{16}$. Considering the right triangle formed by dropping the perpendicular from X to the midpoint M of isosceles triangle $\triangle AXD$, we get $DM = \sqrt{79}$ so $DX = \sqrt{79} \cdot \frac{16}{9}$. Finally, computing $XY = DY - DX$, we get $XY = \boxed{\dfrac{11\sqrt{79}}{9}}$. □