Instructions:

You will have 2 hours to complete the EDAMAME. All answers must be fully simplified; fractions must be reduced to lowest terms, and square factors must be moved outside radicals. Decimals are accepted provided they are exact. You may **NOT** use rulers or calculators. You may only use pens, pencils, blank paper, erasers, and compasses. Scrapwork **will be collected**. Write your name on each piece of scrap paper you use. Please write your full name and answers clearly and legibly.

Name: ______________________________

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Problem 1. The polynomial $x^2 + 5x + 1$ has roots a and b. Evaluate $\left(a + \frac{1}{a}\right)\left(b + \frac{1}{b}\right)$.

Problem 2. Compute the least positive integer n such that n^2 has an even number of digits and has 999 as its leftmost three digits.

Problem 3. Takaki writes down a two digit number (possibly with a leading zero) to send to Akari. Then, James, Kyle, and Sophia take turns randomly choosing one of the digits and changing it to a randomly chosen different digit. When Sophia finishes her turn, she sends the number to Akari. What is the probability that Akari gets the original number?

Problem 4. Let $ABCD$ be a unit square with point F on BC and point E on AD, and let X denote the intersection of EF and AC. If $AE = 2CF$ and $BX \perp EF$, compute the length of segment CF.

Problem 5. Let k be a real number such that the polynomial $p(x) = x^3 - 6x^2 + 36x - k$ has three distinct roots z_1, z_2, and z_3 which form a nondegenerate right triangle when plotted in the complex plane. Find the sum of all possible values of k.

Problem 6. Let a, b be positive integers with $a > b \geq 3$. A regular a-gon and b-gon sharing a vertex are inscribed in the same circle and their vertices and the intersections between their sides are marked. If the total number of marked points is 35, find the sum of all possible values of a.

Problem 7. One person is standing in each cell of a 4×4 grid. How many ways are there for each person to move to an orthogonally adjacent cell such that each cell is occupied afterwards and no pair of people swapped places?

Problem 8. Triangle $\triangle ABC$ with $AB = AC = 6$ and $BC = 10$ is inscribed in circle Ω. A point D is on Ω with $AD = 9$. Find the area of quadrilateral $ABDC$.

Problem 9. Given that p is a prime number and

$$\frac{1}{p} = 0.\overline{0002a240609997175b39}$$

for some unknown digits a and b, compute p.

Problem 10. Complex numbers a, b, and c satisfy the equations

$$\begin{aligned} ab + b + c + 1 &= 0 \\ bc + c + a + 1 &= 0 \\ ca + a + b + 1 &= 0. \end{aligned}$$

Find the sum of all possible values of a^2.

Problem 11. On the coordinate plane, Elma is at $(0, 0)$ and Visby is at $(32, 32)$. Each minute, Elma chooses to either move one unit to the right or one unit upwards with equal probability. She then moves in that direction if and only if her coordinate in that direction is less than 32 (for example, if she is at the point (4, 32) and chooses to go upwards, she doesn't move that minute). Compute the expected number of minutes that Elma will not move before she reaches Visby.

Problem 12. Evaluate the sum

$$\sum_{n=0}^{\infty} \frac{\sin^4\left(\frac{2^n \pi}{101}\right)}{4^n}.$$

Problem 13. Let $\triangle ABC$ be a triangle with $AB = 8$, $AC = 10$, and $BC = 12$. Point D is chosen on line BC such that B is between D and C. Let Ω_1 be circumcircle of $\triangle ABC$ and Ω_2 be circumcircle of $\triangle ABD$. Let tangents to Ω_2 at A and Ω_1 at B meet at X, and the tangent to Ω_2 at D and tangent to Ω_1 at C meet at Y. If D, X, Y are collinear, compute the length XY.