Set 1

1. [**5**] Two line segments $\overline{BR}$ and $\overline{UM}$ intersect at a point O such that $BO \times RO = UO \times MO$. Find $\angle BUR + \angle RMB$.

2. [**5**] A 5-digit number contains no repeating digits. If the product of its digits is 5670, find the sum of its digits.

3. [**5**] A scientist drops a ball from a height of 243 centimeters. Each time it bounces back up, it reaches a maximum height $\frac{2}{3}$ of what it was before. Eventually, the ball's peak is below 40 centimeters, and the now bored scientist walks away to make a sandwich. Right as it reaches that last observed peak (which is below 40 centimeters), how much distance has the ball traveled?

. . . .

Set 2

4. [**6**] Find the sum of the coefficients of $(xy + 2x + 3y)^4$.

5. [**6**] Bruno the bear sits in a circle with his friends Alice, Bob, Charlie, David, and Eve. If Bruno refuses to sit next to Bob, how many ways are there to seat the group of 6 friends? (Rotations of a seating arrangement are considered to be the same.)

6. [**6**] A point on the xy-plane is called "nice" if both of its coordinates are rational numbers. Given a convex 2025-gon P inscribed in a circle O whose center is NOT "nice", what is the maximum number of "nice" vertices of P?

. . . .

Set 3

7. [**7**] Let $h(n)$ be defined as the greatest n-digit number, all of whose digits are prime, that is the product of n distinct prime numbers. Compute $h(4)$.

8. [**7**] a is chosen randomly on the interval $[0, 4]$, and b is chosen randomly on the interval $[0, 2]$. What is the probability that $1 \le |a - b| \le 2$?

9. [**7**] Consider a room shaped like a hexagonal pyramid with base side length 2 and height $2\sqrt{6}$. A beetle is currently at the centroid of one of the walls. If it may only crawl among the walls and floor of the room, how long is its shortest path to one of the base vertices of the wall opposite to the one it's on right now?

. . . .

Set 4

10. [**8**] How many ways can you write $19 = a + b + c + d + e$, where a, b, c, d, and e are distinct positive integers?

11. [**8**] Let $a_1, a_2, a_3, \ldots$ be a geometric sequence of real numbers with $a_n \neq 0$ for all positive integers n. Given that $a_3 = 1$, find the minimum possible value of $a_1 + 2a_2 + 2a_4 + a_5$.

12. [**8**] Let P be a point in the interior of $\triangle ABC$. Let D be a point on BC such that the circumcircles of $\triangle CDP$ and $\triangle BDP$ intersect AC and AB at E and F, respectively. Suppose that EF is tangent to both circumcircles. Given that $\angle A = 55°$, compute $\angle BPC$ in degrees.

Set 5

13. [**9**] Find the product of all real numbers $x > 0$ for which

$$\log_2\left(\frac{1}{x^2}\right) + \log_x(8) = \left(\frac{5}{\log_x(4) + 3\log_2(x)}\right).$$

14. [**9**] Define a function $A(n)$ below for all integers n

$$A(n) = \begin{cases} A(A(n+5)) & \text{if } n \leq 1000 \\ n-2 & \text{if } n > 1000 \end{cases}$$

What is the value of $A(5)$?

15. [**9**] We fill each cell of an 8×8 table with a number from $\{1, 2, 3, 4, 5, 6\}$. How many ways are there to fill the table such that the sum of numbers in each row is divisible by 2 and the sum of numbers in each column is divisible by 3?

. . . .

Set 6

16. [**10**] Let the arithmetic and harmonic means of the areas of three distinct similar triangles be 42 and $\frac{96}{7}$, respectively. Given that the second largest triangle has area and perimeter 24, find the arithmetic mean of the perimeters of the three triangles.

17. [**10**] Let $ABCD$ be an isosceles trapezoid with $AB = 12, CD = 28$, and $AD = BC = 17$. Let Ω be the circle with diameter BC, and let $\overline{AC}$ intersect Ω at $K \neq C$. Compute AK.

18. [**10**] A box has 11 slips of paper in it each labeled with a different integer between 1 and 11, inclusive. Carla picks four random slips of paper from the box, without replacement, and then arranges them in ascending order. What is the probability that, once she has finished arranging the slips of paper, two consecutive slips have a difference of exactly 5?

. . . .

Set 7

19. [**11**] Suppose $x^3 - 5x + 2 = 0$ has 3 distinct complex roots a, b, c. Compute $a^4 + b^4 + c^4$.

20. [**11**] Define $p(x) = (1+x+x^2+\cdots+x^{203})^3$. Expand $p(x)$ into the form $a_0+a_1x+a_2x^2+\cdots+a_{609}x^{609}$. Find a_{300}.

21. [**11**] Choose six distinct vertices of a regular nonagon. What is the expected number of distinct equilateral triangles that can be drawn using only the chosen vertices?

Set 8

22. [**12**] There is a classroom with chairs arranged in 4 rows of 6 chairs facing the front of the classroom. Bruno wants to make a seating chart for his 23 students. However, he is worried that students might be blocked by taller students sitting in chairs in front of the student, in the same column of chairs.

    If Bruno selects a seating arrangement uniformly at random, what is the expected number of students who won't be blocked by a taller one? Here, assume that each student has a unique height.

23. [**12**] Given that a and b are non-negative numbers and $3a + b = 10$, find the minimum value of $\sqrt{9a^2 + 16} + \sqrt{b^2 + 9}$.

24. [**12**] Let ABC be a triangle with $AB = 1$. Let l be the line passing through A parallel to $\overline{BC}$. Suppose that there is a circle passing through C tangent to $\overline{BC}$, $\overline{AB}$, and l. Compute the maximum possible area of $\triangle ABC$.

. . . .

Set 9

25. [**13**] Let a, b, and c be distinct real numbers such that

$$a = \frac{bc + 2}{1 - b - c}$$
$$b = \frac{2ac + 1}{1 - 2a - 2c}$$
$$c = \frac{3ab + 4}{1 - 3a - 3b}.$$

    Compute $a + b + c$.

26. [**13**] Nijika has a fair n sided die with $n \geq 4$, with each side containing one of the distinct integers from 1 to n inclusive. The average number of times she needs to roll for her to get m consecutive rolls of the same integer, with $m \geq 1$, is $97 - 3n$. What is the sum of all possible values of n?

27. [**13**] Let ℓ be a line with slope less than -1 and positive y-intercept which is tangent to the hyperbola $y = 1/x$. Let Δ denote the triangle bounded by ℓ, the x axis, and the y axis. Let ℓ' denote the line obtained by reflecting ℓ across the line $y = x$. If ℓ' splits Δ into two regions of equal area, what is the x-intercept of ℓ?

. . . .

Set 10

28. [**14**] Let $\tau(n)$ be the number of divisors of n. Call an integer m a *tower* if $a^{\tau(a)} \not\equiv 1 \pmod{m}$ holds for all $2 \leq a \leq m - 1$. Given that there are five towers between 3 and 100 (inclusive), compute the sum of the five towers.

29. [**14**] Let z be a complex number. Find the minimum possible value of $|z^2 - 4z + 9| + |z^2 + 4z + 9|$.

30. [**14**] Call an integer n *worthless* if the sum of its positive divisors (including n itself) is strictly less than $\frac{8n}{7}$. How many positive integers less than or equal to 1050 are worthless?

Set 11

31. [**15**] The Fibonacci sequence F_n is defined as $F_0 = 0$, $F_1 = 1$, and $F_{n+2} = F_{n+1} + F_n$ for $n \geq 0$. Consider the polygon with vertices at Cartesian coordinates $(0, 0), (F_0, F_1 + 1), (F_1, F_2 + 1), (F_2, F_3 + 1), \ldots, (F_{10}, F_{11} + 1)$ in order. What is the area of this polygon?

32. [**15**] Yor and Loid are playing a game with a set of n cards, numbered with the integers from 1 to n. Yor knows the value of n, but Loid does not.

    Yor, with 2 of the cards already in her hand, tells Loid, "I know that the full deck consists of n cards. If you draw 2 cards, there is a $\frac{1}{30}$ chance that all four of our cards will sum to 45.

    Loid draws two cards, and tells Yor, "Based on my cards, now I know there are two possible values for n."

    Let Loid's highest card be s. What is the sum of all possible values of s?

33. [**15**] Let $ABCDEFGHIJKL$ be a regular dodecagon with side length 1. Points W, X, Y, and Z are chosen uniformly at random from line segments AB, CD, EF, and GH respectively. What is the expected value of the area of quadrilateral $WXYZ$?

. . . .

Set 12

34. [**20**] Let $f(x)$ be a sixth-degree polynomial with no quadratic term satisfying $f(n) = n^{10}$ for $n \in 0, 1, 5, 6, 7, 8$. Estimate $\frac{f(100)}{10^{10}}$. If your estimate is E and the answer is A, you will receive $\max(0, \lfloor 20 \min(\frac{E}{A}, \frac{A}{E})^{0.6} \rfloor)$ points.

35. [**20**] We say a positive integer n is *sweet* if it satisfies both of the following properties:

    - It has at least 6 divisors.
    - Let $d_1 < d_2 < \ldots < d_k$ be the divisors of n. Then for all i such that $3 \leq i \leq k$, we have $d_i d_{i-2} \mid n$ or $n \mid d_i d_{i-2}$ (or both).

    Estimate the number of sweet integers between 1 and 10^6 (inclusive). If your estimate is E and the answer is A, you will receive $\max\left(0, \left\lfloor 20 - \left(\frac{|A-E|}{200}\right)^{0.6} \right\rfloor\right)$ points.

36. [**20**] Every year, thousands of new papers are published in mathematics — you are tasked with determining the relative number of published papers in the following categories: (A) Number Theory, (B) Probability Theory, (C) Dynamical Systems, (D) Complex Variables, (E) Geometric Topology, (F) Category Theory, and (G) Combinatorics. Based on 2024 data from arXiv.org, write down n of these categories (based on their corresponding letter), in ascending order from least to most papers published in 2024. If these n categories are in the correct relative order and $n \geq 4$, your team will earn $(n - 2) \times (n - 3)$ points. Otherwise, you will earn 0 points.