NYCMT 2025-2026 Homework #1

NYCMT

September 12 - September 19, 2025

These problems are due September 19. Please solve as many problems as you can, and write up solutions (**not just answers!**) to the ones you solve. Write down any progress you make on problems you don't solve.

Please write solutions for different questions on separate pages. Make sure to write your name on each page and page numbers per problem, if there are multiple.

If you're not going to be present on September 19th, you can scan your solutions and email them to jamespapaelias@gmail.com, kylewu32@gmail.com, and sjschool26@gmail.com. If you will be there, just hand in your responses on paper. If you have any questions, feel free to ask one of us on Discord or use one of the above emails.

Problems are NOT difficulty-ordered, so you should read and try all of them.

Enjoy!

Problem 1. Let $a_1, a_2, a_3, \ldots$ be an infinite sequence of positive integers with the property that the product of any set of $k \geq 1$ consecutive terms is not a 2025th power. Prove that there exist infinitely many primes which divide some element of the sequence $\{a_i\}$.

Problem 2. Triangle ABC has $AB = 10$, $BC = 17$, and $CA = 21$. Point P lies on the circle with diameter AB. What is the greatest possible area of APC?

Problem 3. A positive integer n is called *internet-enabled* if the binary representation of n^2 contains exactly two 1's. Find the sum of the first 5 *internet-enabled* numbers.

Problem 4. Let $ABCD$ be an isosceles trapezoid with $AB \parallel CD$ and $BC = AD = 10$ and $CD = 20$. Let Ω be the circumcircle of $ABCD$, and let M be the midpoint of AB. Line CM intersects Ω at $X \neq C$ and DX intersects AB at Y. Given that $AY = 3$, compute the length of side AB.

Problem 5. Let $n \geq 2$ be a positive integer and let $z_1, \ldots, z_n$ be nonzero complex numbers satisfying $\overline{z_k} + \frac{1}{z_k} = 2z_{k+1}$ for each $1 \leq k \leq n$, where indices are taken cyclically. Find, in terms of n, all possible values of $(z_1, z_2, \ldots, z_n)$.