

# NYCMT 2024-2025 Homework #4

NYCMT

January 3 - January 17, 2024

These problems are due January 17. Please solve as many problems as you can, and write up solutions (**not just answers!**) to the ones you solve. Write down any progress you make on problems you don't solve. Please write solutions for different questions on separate pages. Make sure to write your name on each page and page numbers per problem.

If you're not going to be present on January 17, you can scan your solutions and email them to [ashleyzhu111@gmail.com](mailto:ashleyzhu111@gmail.com), [sjschool26@gmail.com](mailto:sjschool26@gmail.com), and [stevenyt-lou@gmail.com](mailto:stevenyt-lou@gmail.com). If you will be there, just hand in your responses on paper. If you have any questions, feel free to ask one of us on Discord or using one of the above emails.

Problems are NOT difficulty-ordered, so you should read and try all of them. Enjoy!

**Problem 1.** Prove that if  $n$  is a nonnegative integer, then  $19 \cdot 8^n + 17$  is not a prime number.

**Problem 2.** Find all positive integers  $n \geq 2$  for which one can fill in the cells of an  $n \times n$  grid with the numbers 0, 1, 2 such that, when calculating the sum of the numbers in each row and each column, the numbers  $1, 2, \dots, 2n$  are obtained in some order.

**Problem 3.** Let  $ABCD$  be a cyclic quadrilateral such that no two sides are parallel to each other. Let lines  $AB$  and  $CD$  intersect at  $E$ , and let lines  $AD$  and  $BC$  intersect at  $F$ . Prove that the angle bisectors of  $\angle AFB$  and  $\angle BEC$  are perpendicular to each other.

**Problem 4.** Does there exist a sequence of 2025 consecutive positive integers such that the  $k$ th term is divisible by  $2026 - k$  for all  $1 \leq k \leq 2025$ ?

**Problem 5.** Let  $P(x)$  be a polynomial with degree at most 8 such that for  $k = 0, 1, \dots, 8$ ,

$$P(k) = \begin{cases} 0 & k \equiv 0 \pmod{3} \\ 1 & k \equiv 1 \pmod{3} \\ 2 & k \equiv 2 \pmod{3}. \end{cases}$$

Find  $P(9)$ .