Instructions:

- You will have 2 hours to complete the CANDLE.
- All answers must be fully simplified; fractions must be reduced to lowest terms, and square factors must be moved outside radicals.
- Decimals are accepted provided they are exact.
- You may **NOT** use rulers, compasses, or calculators. You may only use pens, pencils, blank paper, and erasers.
- Scrapwork will be **collected**. Write your name on each piece of scrap paper you use.

Please write your full name and answers clearly and legibly.

Name: ______________________________

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Problem 1. If $x+y+z \equiv 20 \pmod{41}$ and $x^2+y^2+z^2 \equiv 33 \pmod{41}$, compute $(x+y)^2+(x+z)^2+(y+z)^2 \pmod{41}$.

Problem 2. Six people are sitting at a round table in a cafe. Each one orders a cookie or a brownie with equal probability. What is the probability that no two adjacent people order brownies?

Problem 3. Find the sum of real x such that $0 \leq x < 2\pi$ satisfying the equation $\cos x \cdot \cos 2x \cdot \cos 4x \cdot \cos 8x = \frac{1}{32 \sin x}$.

Problem 4. Let $\triangle ABC$ be a triangle with $\angle A = 60°$, $AB = 5$, $AC = 7$. Let the bisector of $\angle BAC$ meet the circumcircle of ABC again at D. Find the area of quadrilateral $ABDC$.

Problem 5. Find all integer k such that $\frac{k^3+29k}{k^2-1}$ is also an integer.

Problem 6. Sophia rolls five six-sided dice and sums the numbers that come up. What is the probability that the result is a multiple of 7?

Problem 7. Let $\triangle ABC$ be a triangle where $\angle B$ is obtuse. Let ℓ be the external angle bisector of A, and let ℓ intersect line BC at D and the circumcircle of ABC at $E \neq A$. Let P be the intersection of AC and BE. If $AC = BE = 45$ and $AB = 9$, what is the length of DP?

Problem 8. Steven's locker combination is an ordered triple of integers (a, b, c), such that $1 \leq a, b, c \leq 20$. He forgot his exact combination, but he remembers that the $\gcd(a, b, c) = 1$. How many possible locker combinations could Steven have?

Problem 9. For real numbers x and y,

$$(1-xy)^2 = \frac{4}{13}(1+x^2)(1+y^2)$$
$$(x+1)(y+1) = 12.$$

Find all ordered pairs of solutions (x, y).

Problem 10. Let N be the number of functions

$$f : \{1, 2, 3, \ldots, 12\} \mapsto \{1, 2, 3, \ldots, \operatorname{lcm}(1, 2, 3, \cdots, 12)\}$$

such that all integers a, b such that $1 \leq b < a \leq 12$ satisfy $a - b \mid f(a) - f(b)$. Find $\tau(\tau(N))$, where $\tau(k)$ denotes the number of positive integer divisors of k.

Problem 11. Let $\triangle ABC$ be a triangle with $AB = 15$, $AC = 13$, and $BC = 14$. Points K and L are on the same side of $\overline{BC}$ as A, and satisfy $\angle ABK = \angle ABC$ and $\angle ACL = \angle ACB$. Lines $\overline{KB}$ and $\overline{LC}$ intersect at D. If O is the circumcenter of $\triangle ABC$, and the length of OD can be expressed as $\frac{a}{b}$, find $a + b$.

Problem 12. The sequence $a_0, a_1, \ldots$ is defined by

$$\sum_{i=0}^{n} (-1)^i \binom{n}{i} a_{n-i} = n^3$$

for all nonnegative n. If $\tau(a_{2021})$ is the number of divisors of a_{2021}, find the remainder of $\tau(a_{2021})$ when divided by 1000.