2018 Fall Stuyvesant Team Contest

1. [**5**] Stan flips an unfair coin. Given this information, what is the probability that it comes up heads?

   Team Name: _____ Answer: _____

2. [**5**] Find the least positive integer n such that the second digit of 11^n is not n.

   Team Name: _____ Answer: _____

3. [**5**] The number 2018 has the property that its first digit and last digit sum to a number which is represented by the reverse of the other digits; that is, $2 + 8 = 10$. What is the next positive integer with this property?

   Team Name: _____ Answer: _____

4. [**6**] Compute the number of proper divisors of 1023.

   Note: A positive integer m is a proper divisor of a positive integer n if $\frac{n}{m}$ is an integer greater than 1.

   Team Name: _____ Answer: _____

5. [**6**] Find the least positive integer n such that a regular n-gon has interior angles of greater than $179°$.

   Team Name: _____ Answer: _____

6. [**6**] A Stuyvesant student sleeps at 11:59 pm and wakes up at 12:01 am. How many times during a Stuyvesant student's day (while they are awake) do the minute hand and hour hand of a clock line up?

   Team Name: _____ Answer: _____

7. [**7**] If $x + y = 7$ and $\frac{1}{x} + \frac{1}{y} = 0.7$, compute the greater of the two values x and y.

   Team Name: _____ Answer: _____

8. [**7**] Compute the sum of all x satisfying $4^x + 128 = 3 \cdot 2^{x+3}$.

   Team Name: _____ Answer: _____

9. [**7**] How many ways can Kimi get from the second floor to the fourth floor using any combination of the three staircases and the two relevant escalators? (He may only go up and across floors)

   Team Name: _____ Answer: _____

10. [**8**] Let $s(n)$ denote the sum of the digits of n, and let $f(n) = 11s(n) - n$. For how many two digit numbers n is $f(n)$ also a two digit number?

    Team Name: _____ Answer: _____

11. [**8**] Compute the number of integers n such that $2 \leq n \leq 2018$ and $\binom{n}{2}$ is relatively prime to 2018.

    Team Name: _____ Answer: _____

12. [**8**] If a regular 2018-gon $A_1A_2\cdots A_{2018}$ has area 2018, compute $[\triangle A_1A_2A_{1010}]$.
Note: $[\cdots]$ denotes the area of the polygon in the brackets.
Team Name: _____ Answer: _____

13. [**9**] Positive reals a and b satisfy
$$\sqrt{ab} = \sqrt{a} + \sqrt{b} + \sqrt{a+b}$$
Compute $\sqrt{ab} - 2\sqrt{a+b}$.
Team Name: _____ Answer: _____

14. [**9**] A random positive integer less than or equal to 900 is chosen. What is the probability that the quotient when divided by 30 is greater than the remainder when divided by 30?
Team Name: _____ Answer: _____

15. [**9**] The polynomial $x^3 + 4x^2 + bx + c$ has roots r, s, and t. If $r^2 + s^2 + t^2 = 16$, compute bc.
Team Name: _____ Answer: _____

16. [**10**] How many three digit multiples of 9 have two of the digits summing to the third?
Team Name: _____ Answer: _____

17. [**10**] In $\triangle ABC$, $AB = 13$, $BC = 14$ and $AC = 15$. Let M_1 and M_2 be the trisection points of BC and consider the circles with diameters AM_1 and AM_2. Suppose the circles intersect again at a point $X \neq A$. Compute AX.
Team Name: _____ Answer: _____

18. [**10**] Suppose that $P(x)$ is a cubic polynomial satisfying $P(1) = 4$, $P(2) = 9$, and $P(3) = 16$. Compute $P(4) + P(0)$.
Team Name: _____ Answer: _____

19. [**11**] Two chords of a circle have length 12 and 13. They intersect, forming an angle of 30°. If one of the chords bisects the other, compute the radius of the circle.
Team Name: _____ Answer: _____

20. [**11**] Let A be an arithmetic sequence. Compute the smallest positive common difference d such that 6 consecutive terms of A are primes.
Team Name: _____ Answer: _____

21. [**11**] Compute the sum of all integers n for which
$$\frac{n^4 + n^2 + 400}{n^2 + n + 1}$$
is also an integer.
Team Name: _____ Answer: _____

22. [**12**] How many ways can the squares of a 2 by 3 board be filled with elements of $\{-1, 0, 1\}$ if adjacent squares cannot sum to 0?

Team Name: _____ Answer: _____

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23. [**12**] Positive real numbers a, b, c, d satisfy

$$a^2 - \sqrt{3}ab + b^2 = c^2 + \sqrt{3}cd + d^2 = 81$$

Find the maximum value of $ac + bd$.

Team Name: _____ Answer: _____

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24. [**12**] How many ordered triples of integers (a, b, c) satisfy $1 \leq a, b, c \leq 30$ and $(1+2a)(1+3b)(1+5c)-1$ is divisible by 30?

Team Name: _____ Answer: _____

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