

NYCMT 2024-2025 Homework #2

NYCMT

September 27 - October 18, 2024

These problems are due October 18. Please solve as many problems as you can, and write up solutions (**not just answers!**) to the ones you solve. Write down any progress you make on problems you don't solve. Please write solutions for different questions on separate pages. Make sure to write your name on each page and page numbers per problem.

If you're not going to be present on October 18, you can scan your solutions and email them to ashleyzhu111@gmail.com, sjschool26@gmail.com, and stevenyt-lou@gmail.com. If you will be there, just hand in your responses on paper. If you have any questions, feel free to ask one of us on Discord or using one of the above emails.

Problems are NOT difficulty-ordered, so you should read and try all of them. Enjoy!

Problem 1. There are 2024 math teamers sitting around at a round table. Combo mains always lie, Geo mains always tell the truth, and Alg mains randomly tell the truth or lie. Given that all math teamers are either Combo, Geo, or Alg mains, there is at least one of each type at the table, and you are only allowed to ask yes/no questions to a math teamer regarding the identity of another, is it possible to always identify all 2024 math teamers?

Problem 2. Find the number of ordered quadruples of positive integers (a, b, c, d) satisfying $a! \cdot b! \cdot c! \cdot d! = 24!$.

Problem 3. Let $R(x)$ be the remainder polynomial obtained when x^{2024} is divided by $x^2 - 3x + 2$. Find the value of $R(3)$.

Problem 4. Let ω be a circle centered at a point O , and $P \neq O$ be a point inside ω . Let A be a point on ω , and X be the point on \overline{OA} such that $XA = XP$. Show that as A moves along ω , X moves along an ellipse.

Problem 5. Find all ordered positive integer 3-tuples (a, b, c) such that

$$\frac{1}{a+b-c} + \frac{1}{a+c-b} + \frac{1}{b+c-a} = \frac{1}{12}$$
$$\frac{c}{a+b-c} + \frac{b}{a+c-b} + \frac{a}{b+c-a} = 4$$