SHRIMP

Seasonal Holiday Rush Invitational Math Playoff

1. Andrew and Jaemin are expert painters, and each of them can create one painting in five days. However, when working together, they distract each other, with Andrew working at half of his normal pace, and Jaemin working at a third of his normal pace. How many days would it take the two of them, working together, to create one painting? (Assume Andrew and Jaemin work at constant rates).

2. Taylor Swift has 4 spaces in a row on which she can place her 10 indistinguishable cookies. If every cookie must be placed on one of the 4 spaces, and any two spaces with cookies must have at least one blank space *baby* between them (meaning two spaces with cookies cannot be adjacent), in how many possible ways can she place her cookies?

3. The numbers $1, 2, 3, \ldots, 7$ are partitioned into two groups. Let the product of the numbers in the first group be A and the product of the numbers in the second group be B. Find the minimum possible value of $|A - B|$.

4. Square $ABCD$ has area 16. Point P is chosen in the plane such that $[PAB] = 4$, $[PCD] = 12$, and $PA = PC = x$. Find x.

5. The ordered pair of positive integers (x, y) satisfies
   - $\gcd(x, y) = 11$
   - $\gcd(2023x, y) = 187$

   Find the smallest possible value of $x + y$.

6. A circle of radius 5 is inscribed in an isosceles trapezoid. The distance between the tangency points of the circle with the legs of the trapezoid is 8. Find the area of the trapezoid.

7. p and q are primes such that $p+q$ is prime and p^2+q is prime. Compute all possible values of p^3+q.

8. Suppose a monic polynomial $f(x)$ of degree 3 satisfies the equation $f(2^n-1)=4^n-1$ for $n=1,2,3$. Find $f(0)$.

9. The numbers $1,2,\ldots,8,9$ are placed randomly into a 3×3 grid such that each number appears exactly once. What is the probability that each 2×2 square contains at least one perfect square?

10. A sequence a_n of integers is defined by $a_1=2$, $a_2=4$ and, for all integers $n\geq 3$,

$$\frac{1}{a_1}+\frac{1}{a_2}+\ldots+\frac{1}{a_{n-1}}+\frac{n}{a_n}=1.$$

Find $a_{2023}-a_{2022}$.

11. A sequence $a_0,a_1,\ldots a_9,a_{10}$ of non-negative integers is called *slow* if, for all $0\leq k\leq 9$, $|a_{k+1}-a_k|\leq 1$. How many *slow* sequences are there such that $a_0=0$ and $a_{10}=6$?

12. Let $P(x)$ be a monic cubic polynomial with integer coefficients. Suppose it has roots a, b, and c that satisfy:

$$3=\frac{(a+b)(b+c)(c+a)}{10}=\frac{1}{a}+\frac{1}{b}+\frac{1}{c}$$

Find the sum of all possible values of $|P(1)|$.

13. Let $f(x)=x^2+nx+6$, with n being a positive integer. Suppose there exist exactly 3 real solutions to the equation $f(f(f(x)))=f(x)$. Find n.

14. Suppose we have $\triangle ABC$ with centroid G and altitude $AH=6$ with point H on side $\overline{BC}$ such that $BH=2$. Let point D be the intersection of the angle bisector of $\angle AHB$ and side $\overline{AB}$, and point E be the intersection of $\overline{AC}$ and the extension of $\overline{DG}$ through G. If $m\angle DHE=90°$, find DE.

15. Suppose four distinct points are chosen from the vertices of a regular 24-gon to form a convex quadrilateral. How many different quadrilaterals are there with all angles less than $135°$? (Two quadrilaterals that are rotations of each other are considered different).