NYCMT 2023-2024 Homework # 5

Quadrilateral $ABCD$

February 9th - March 1st, 2024

These problems are due March 1st. Please solve as many problems as you can, and write up solutions (**not just answers!**) to the ones you solve. Write down any progress you make on problems you don't solve. Please write solutions for different questions on separate pages. Make sure to write your name on each page and page numbers per problem.
If you're not going to be present on March 1st, you can scan your solutions and email them to ali40@stuy.edu, dpotievsky40@stuy.edu, and jaeminkim2@hunterschools.org. If you will be there, just hand in your responses on paper. If you have any questions, feel free to ask one of us on Discord or using one of the above emails.
Problems are NOT difficulty-ordered, so you should read and try all of them. Enjoy!

Problem 1. Quadrilateral $ABCD$ has diagonals with integer lengths and positive area. If $AB = AD = 10$ and $CB = CD = 17$, find the smallest possible value of its area.

Problem 2. Quadrilateral $ABCD$ is a 20×24 rectangular grid of cells containing 9 mines that are randomly placed, with at most one mine per cell. A cell contains a number denoting the number of touching cells that contain a mine if and only if it does not contain a mine. Let P_8 be the probability that the grid contains an 8, and let P_7 be the probability that the grid contains a 7. Find $\frac{P_7}{P_8}$.

Problem 3. Quadrilateral $ABCD$ is graphed on the complex plane with $A = z_1$, $B = z_2$, $C = i$, and $D = z_3$, satisfying the equation $5z_1 + i = 3z_2 + 3z_3$. If A, B, and D all lie on the circle centered at C with radius 6, find $[ABCD]$.

Problem 4. Quadrilateral $ABCD$ has integer side lengths such that the sum of any three is a multiple of the fourth. Show that the side lengths cannot all be distinct.

Problem 5. Quadrilateral $ABCD$ is a parallelogram with point P in its interior such that $\triangle PAB$ is equilateral. Let the midpoint of $\overline{AD}$ be M_1 and the midpoint of $\overline{CP}$ be M_2. Prove that $\overline{M_1M_2}$ is perpendicular to $\overline{BP}$.