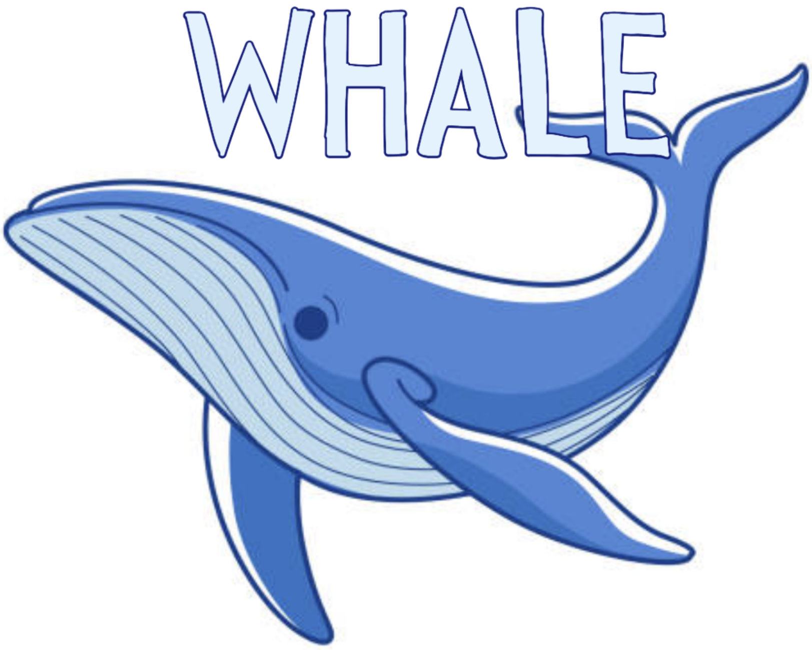


**We're Having Another Legitimate Exam!**

**Instructions:**

- You will have 2 hours to complete the WHALE.
- All answers are integers between 0 and 999 inclusive.
- You may **NOT** use rulers, compasses, or calculators. You may only use pens, pencils, blank paper, and erasers.

1. Let $n$ be the smallest positive perfect square such that $2024n$ is a perfect cube. Compute $\sqrt{n}$.

2. An Olympic-size swimming pool is 50 meters long and can hold up to 2500 cubic meters of water. Daniel is building a to-scale model that is 0.1 meters wide and 0.008 meters deep. How much water can Daniel's model hold, in cubic centimeters?

3. A string of letters is called *fruity* if there exists a pair of consecutive letters that are the same. For example, APPLE is fruity, but BANANA is not. The word GRAPEFRUIT is written on a chalkboard. Andrew erases a random letter each minute until the chalkboard is blank. The probability that the string of letters on the chalkboard is never fruity can be expressed as $\frac{m}{n}$ where $m$ and $n$ are relatively prime positive integers. Find $m+n$.

4. Find the smallest prime $p$ such that $p^2 + 2024$ has 27 divisors.

5. Point $P$ lies on diameter $\overline{XY}$ of a circle with radius 4. Chord $\overline{AB}$ passes through $P$ and makes a $30^\circ$ degree angle with $\overline{XY}$. Let $C$ be the reflection of $B$ over $\overline{XY}$. If $\frac{AP}{PB} = \frac{2}{3}$, then the area of $\triangle ABC$ can be expressed as $\frac{a\sqrt{b}}{c}$, where $a$ and $c$ are relatively prime positive integers and $b$ is a squarefree integer. Find $a+b+c$.

6. Find the unique integer $k$ such that the polynomial $x^3 - 14x^2 + 62x - k$ has three zeroes that are the side lengths of a right triangle.

7. Three standard six-sided dice are rolled, and let $A$ be their sum. Then, the die with the lowest number is re-rolled. (If there are multiple dice with the lowest number, only one of them is re-rolled.) Let $B$ be the sum of the dice after the re-roll. The probability that $B > A$ can be expressed as $\frac{n}{6^4}$. Find $n$.

8. How many ways are there to color the faces of a cube one of five colors such that no two faces sharing an edge are the same color? Rotations are considered distinct.

9. Square $ABCD$ has $E$ as the midpoint of $\overline{AB}$. Let $P$ be the point on $\overline{BC}$ such that the line $\overleftrightarrow{PD}$ intersects $\overline{EC}$ and $\overline{AC}$ at $F$ and $G$ respectively, and $\frac{AG}{GC} = \frac{CF}{FE}$. If $\frac{CP}{PB}$ can be expressed as $\frac{a+\sqrt{b}}{c}$, where $a$ and $c$ are relatively prime positive integers and $b$ is a squarefree integer, find $a+b+c$.

10. Consider a sequence of non-negative integers defined with $x_1, x_2 < 1000$, and $x_k = \min\{|x_i - x_j|, 0 < i < j < k\}$ for all integers $k \geq 3$. For example, $x_3 = |x_1 - x_2|$ and $x_4 = \min\{|x_1 - x_2|, |x_1 - x_3|, |x_2 - x_3|\}$. Find the greatest possible value of $x_{17}$ over all such sequences.

**Note that problems** 11 **and** 12 **are on the next page**.

11. In isosceles $\triangle ABC$ with $AB = AC = 10$, circles $\omega_1$ and $\omega_2$ are centered at $B$ and $C$ with radii 6 and 8, respectively. Point $G_1$ is on $\omega_1$ such that $AG_1 = \frac{32}{3}$ and $\overline{AG_1}$ intersects $\omega_1$ again at point $F_1$. Point $G_2$ is on $\omega_2$ such that $AG_2 = 8$ and $\overline{AG_2}$ intersects $\omega_2$ again at point $F_2$. If the circumradius of $\triangle AF_1F_2$ is 4, the positive difference between the maximum and minimum value of $G_1G_2^2$ can be expressed as $\frac{m}{n}$, where $m$ and $n$ are relatively prime positive integers. Find $m + n$.

12. Andrew loves painting whales. There are 100 blue whales floating in a circle. An integer $n$ is chosen with $2 \leq n \leq 50$. In a given move, Andrew chooses a set of $n$ consecutive whales that are floating adjacently, with the first and the last whales being blue, and paints the first and last whales white. Find the sum of all values of $n$ for which Andrew can paint all 100 whales white after 50 moves.