Instructions: You have 120 minutes to complete the problems. All answers must be fully simplified; fractions must be reduced to lowest terms, and square factors must be moved outside radicals. Decimals are accepted provided they are exact.
$[ABC]$ denotes the area of ABC.
You may **NOT** use rulers, compasses, or calculators. You may only use pens, pencils, blank paper, and erasers.
Scrapwork will be **collected**. Write your name on each piece of scrap paper you use.

1. If $a^3 + a - 1 = 0$, find $a^5 - a^2 - a + 6$.
2. How many ordered pairs of integers are there such that the absolute difference between their product and their sum is 35?
3. Point C lies on segment $\overline{AB}$ such that $AC = 8$ and $CB = 10$. Point D lies in the plane such that $\angle ADC \cong \angle DBC$. Find the maximum possible area of $\triangle DAB$.
4. How many ways are there to arrange the numbers $1, 2, 3, \ldots, 9$ in a row such that, for any two integers a and b, if $a \mid b$, then a comes before b?
5. Suppose $\triangle ABC$ has side lengths $AB = 11$, $BC = 16$. and $CA = 13$. Let D be the midpoint of $\overline{BC}$ and E be the point where the angle bisector of A intersects $\overline{BC}$. Let the circumcircle of $\triangle ADE$ meet $\overline{AC}$ and $\overline{AB}$ again at F and G respectively. Suppose $M \neq B$ is a point on $\overline{AB}$ such that $MG = GB$. Find the ratio $\frac{[BME]}{[CAME]}$.
6. Let A be the number of functions $f : \{1, 2, 3, 4, 5, 6, 7, 8, 9\} \longrightarrow \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ such that n and $f(n)$ are different parity and $f(f(n)) \neq n$ for all n. Find the remainder when A is divided by 1000.
7. Find all real x that satisfy the equation $2x\sqrt{1 - x^2} + 2x^2 - \sqrt{2}x - 1 = 0$.
8. Compute the sum of all prime numbers p such that $37p^2 - 47p + 4$ is the square of an integer.
9. Find the number of sequences of integers $(a_0, a_1, \ldots, a_{50})$ with $0 \leq a_n \leq 5$ such that:
$$a_0 + a_1 \cdot 4 + a_2 \cdot 4^2 + \ldots + a_{50} \cdot 4^{50} = 2023 \cdot 4^{45}$$
10. Let $A = \sum_{n=1}^{99} \sqrt{10 + \sqrt{n}}$ and $B = \sum_{n=1}^{99} \sqrt{10 - \sqrt{n}}$. Find $\frac{B}{A}$.
11. Scalene $\triangle ABC$ has side lengths $AB = 20$ and $BC = 23$. If the line connecting the incenter and centroid of the triangle is perpendicular to side AB, find the area of $\triangle ABC$.
12. Define the sequence $a_d = d \cdot\ (d+1) \cdot (d+2)$ for integers $d \geq 1$. Suppose for a positive integer n with $\gcd(n, 6) = 1$, there are 140 values of $k \leq n$ where $\gcd(n, a_k) = 1$. Find the sum of all possible values of n.

Please write your full name and answers clearly and legibly.
You may NOT flip this paper over until instructed to do so.

Name: ______________________________

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