

NYCMT 2023-2024 Homework #4

NYCMT

November 10th - December 1st, 2023

These problems are due December 1st. Please solve as many problems as you can, and write up solutions (**not just answers!**) to the ones you solve. Write down any progress you make on problems you don't solve. Please write solutions for different questions on separate pages. Make sure to write your name on each page and page numbers per problem. If you're not going to be present on December 1st, you can scan your solutions and email them to ali40@stuy.edu, dpotievsky40@stuy.edu, and jaeminkim2@hunterschools.org. If you will be there, just hand in your responses on paper. If you have any questions, feel free to ask one of us on Discord or using one of the above emails.

Problems are NOT difficulty-ordered, so you should read and try all of them. Enjoy!

Problem 1. Find the last two digits of $1^{2023} - 2^{2023} + 3^{2023} - 4^{2023} + \dots + 9^{2023} - 10^{2023}$.

Problem 2. Mr. Sterr and Mr. Kats are both standing on space 1 of a board game with spaces numbered 1, 2, 3...35 in that order. The board loops around, so space 1 is directly after space 35. Both players roll a fair 35-sided die with faces numbered 1 to 35. Then, they each square the number rolled and move forward that many spaces. What is the probability that Mr. Kats is on a higher-numbered space than Mr. Sterr?

Problem 3. Let the sequence a_n be defined recursively as $a_1 = 1$ and $a_{n+1} = (n + 1) \cdot 1000^n + a_n$ for all $n \geq 1$. Find the sum of the digits of $999^2 \cdot a_{2023}$.

Problem 4. How many ways are there to tile a 2×5 board with any number of 1×2 dominoes and 1×4 tetrominoes?

Problem 5. In $\triangle ABC$, $m\angle B = 3m\angle A$, $AC = 22$, and $BC = 10$. Point D is drawn on \overline{AB} such that $m\angle ACD = 2m\angle A$. Find the area of $\triangle ACD$.

Problem 6. This problem is an OPTIONAL puzzle. The answer is a six letter word. Contact Andrew (email: ali40@stuy.edu) to confirm your answer. No solution is required.

Reduce the following numbers:

1. The answer to Problem 1, expressed as a two-digit number, multiplied by 2.
2. The numerator of the answer to Problem 2 (provided the answer is in lowest terms).
3. The answer to Problem 3.
4. The answer to Problem 4.
5. The answer to Problem 5, plus 1.
6. 20232023.