NYCMT 2023-2024 Homework #2

NYCMT

September 29th - October 13th, 2023

These problems are due October 13th. Please solve as many problems as you can, and write up solutions (**not just answers!**) to the ones you solve. Also write down any progress you make on problems you don't solve. Please write solutions for different questions on separate pages. Make sure to write your name on each page and page numbers per problem.
If you're not going to be present on October 13th, you can scan your solutions and email them to ali40@stuy.edu, dpotievsky40@stuy.edu, and jaeminkim2@hunterschools.org. If you will be there, just hand in your responses on paper. If you have any questions, feel free to ask one of us on Discord or using one of the above emails.
Note that problems are NOT difficulty-ordered, so you should read and try all of them. Enjoy!

Problem 1. Andrew and his imaginary friend play a game in which they each start with two fair coins. They take turns tossing a coin of theirs; if it lands heads, they give it to the other person. If tails, nothing happens. If Andrew goes first, and the game ends if either one of the players wins by having all four coins, what is the probability that Andrew loses?

Problem 2. Let $x_1, x_2, ..., x_{2022}$ be the zeros of $f(x) = x^{2023} - 1$ that are not 1. Evaluate

$$\frac{1}{1-x_1} + \frac{1}{1-x_2} + ... + \frac{1}{1-x_{2022}}.$$

Problem 3. Find all solutions in natural numbers to the equation $1! + 2! + ... + x! = y^2$.

Problem 4. A TV logo measuring 3 in. wide and 2 in. high is bouncing around in a TV screen measuring 17 in. wide and 12 in. high. It starts from the top left corner at a 45 degree angle with the sides of the screen. How far will the logo have traveled, in inches, by the time it hits a corner?

Problem 5. Let n be an odd integer. Consider an $n \times n$ grid with either $+1$ or -1 written in each cell. Prove that, having computed the product of the numbers in each row and column, the sum of these $2n$ products cannot be 0.