

ARML Team Questions – 2005

T-1. $ABCD$ is a convex quadrilateral with $AC = BC = 10$ and $AB + 1 = AD = CD = 13$.

Compute the area of $ABCD$.

T-2. Compute the smallest positive integer K such that K and $K + 1$ each have more than four positive factors.

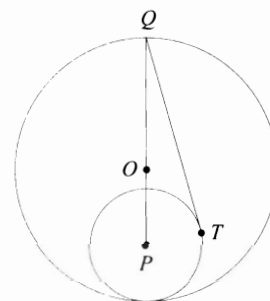
T-3. Let A, R, M , and L be positive real numbers such that $\log(A \cdot L) + \log(A \cdot M) = 2$,
 $\log(M \cdot L) + \log(M \cdot R) = 3$, and $\log(R \cdot A) + \log(R \cdot L) = 4$. Compute the value of the
 product $A \cdot R \cdot M \cdot L$.

T-4. Let a and b be integers with $b > a > 1$. If $8\sqrt{a_b} = \frac{1}{.a_b}$, compute the least value of b .

T-5. Let $N = \overline{.abcdef}$. If N has three 2's in a row in every string $abcdef$, but no more than three 2's in a row in the
 entire decimal expression for N , compute the number of distinct N .

T-6. For x and y in radian measure with $0 \leq x, y \leq 2\pi$, compute the largest possible value of the sum $x + y$
 if $2 \sin x \cos y + \sin x + \cos y = -\frac{1}{2}$.

T-7. In the diagram, circle O has a radius of 10, circle P is internally tangent
 to O and has a radius of 4. \overline{QT} is tangent to circle P at T and, if drawn,
 line \overline{PT} intersects circle O at points A and B . Compute the
 product $TA \cdot TB$.



T-8. For t a real number, let $x(t) = at^3 - bt$ and $y(t) = at^3 + bt^2$ be a system of parametric equations for a
 curve. If $a, b \in \{0, 1, 2, 3, \dots, 100\}$, compute the number of ordered pairs (a, b) such that the curve has
 exactly two distinct y -intercepts.

T-9. Given $A(0, 0)$ and $B(x, y)$ with $0 < x < 1$ and $y > 0$, let the slope of \overline{AB} equal r . Point C lies on the line
 $x = 1$ such that the slope of \overline{BC} equals s with $0 < s < r$. The area of $\triangle ABC$ can be written as $k_x(r - s)$
 where the value of k_x depends on x . Compute the largest possible value for k_x .

T-10. The numbers $1, 2, 3, \dots, 17, 18, 19$ can be written down in a sequence that forms a 29-digit palindrome.
 Compute the number of distinct 29-digit palindromes that can be formed in this way. Write the answer
 without using factorial notation.