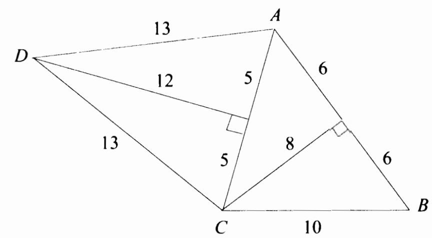
T–1. Noting Pythagorean Triples, we see that the area of ΔACB is $6 \cdot 8 = 48$ and the area of ΔADC is $5 \cdot 12 = 60$, making the area of $ABCD$ equal $\boxed{108}$.



T–2. The simplest numbers that have more than four factors are of the form p^4, p^2q, or pqr where p, q, and r are distinct primes. Trying p^4, we find that 2^4 fails since neither 15 nor 17 has four factors, but 3^4 is a candidate since 80 also has more than four factors. Trying p^2q, $2^2 \cdot 3$ fails since neither 11 nor 13 work, $2^2 \cdot 5$ fails since neither 19 nor 21 has more than four factors, and $2^2 \cdot 7$ fails since 27 and 29 fail. However, $2^2 \cdot 11$ works since 45 has six factors. We also try $3^2 \cdot 2$ but 17 and 19 are prime. Finally, $2 \cdot 3 \cdot 5$ fails since 29 and 31 are prime and $2 \cdot 3 \cdot 7$ fails since 41 and 43 are prime. Answer: $\boxed{44}$.

T–3. From $\log(AL) + \log(AM) + \log(ML) + \log(MR) + \log(RA) + \log(RL) = 2 + 3 + 4$, we obtain $\log\left(A^3R^3M^3L^3\right) = 9 \to \log(ARML)^3 = 9 \to \log(ARML) = 3 \to A \cdot R \cdot M \cdot L = 10^3 = \boxed{1000}$.

<u>Alternate solution</u>: from $\log\left(A^2LM\right) = 2,\ \log\left(M^2LR\right) = 3$, and $\log\left(R^2AL\right) = 4$, we obtain

$A^2LM = 100$, $M^2LR = 1000$, and $R^2AL = 10{,}000$. This gives $\dfrac{M^2LR}{A^2LM} = \dfrac{1000}{100} \to \dfrac{MR}{A^2} = 10$,

$\dfrac{R^2AL}{M^2LR} = \dfrac{10{,}000}{1000} \to \dfrac{RA}{M^2} = 10$. Thus, $\dfrac{MR}{A^2} = \dfrac{RA}{M^2} \to M^3 = A^3 \to M = A$. From

$\dfrac{R^2AL}{A^2LM} = \dfrac{10{,}000}{100} \to \dfrac{R^2}{AM} = 100 \to R^2 = 100A^2$. Thus, $R = 10A$. From $A^2LM = 100$ we obtain

$L = \dfrac{100}{A^3}$. Thus, $A \cdot R \cdot M \cdot L = A \cdot 10A \cdot A \cdot \dfrac{100}{A^3} = 1000$.

T–4. Since $.\overline{a}_b = \dfrac{a}{b} + \dfrac{a}{b^2} + \dfrac{a}{b^3} + \cdots = \dfrac{\frac{a}{b}}{1 - \frac{1}{b}} = \dfrac{a}{b-1}$, we have $8\sqrt{\dfrac{a}{b-1}} = \dfrac{b-1}{a} \to 8 = \left(\dfrac{b-1}{a}\right)^{3/2} \to$

$\dfrac{b-1}{a} = 8^{2/3} = 4 \to b = 4a + 1$. Since $a > 1$, we have $a = 2$ and $b = \boxed{9}$.

T–5. Let x denote any digit except 2 and let y denote any digit. Then the choices for $abcdef$ in which there are three consecutive 2's but no strings of 2's longer than three are the following:

$222xyx$ $\quad x222xy$ $\quad yx222x$ $\quad xyx222$

Each has $9 \cdot 9 \cdot 10 = 810$ possibilities, so there are $4 \cdot 810 = \boxed{3240}$ possible N's.

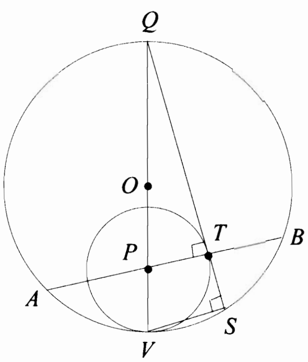
T–6. $2\sin x\cos y + \sin x + \cos y = -\frac{1}{2} \rightarrow 4\sin x\cos y + 2\sin x + 2\cos y + 1 = 0$

$\rightarrow (2\sin x + 1)(2\cos y + 1) = 0 \rightarrow \sin x = -\frac{1}{2}$ and $\cos y$ can be anything from –1 to 1 or $\cos y = -\frac{1}{2}$ and $\sin x$ can be anything from –1 to 1. In the first case, $x = \frac{7\pi}{6}$ or $\frac{11\pi}{6}$ and y could be anything from 0 to 2π. The largest sum is $2\pi + \frac{11\pi}{6} = \frac{23\pi}{6}$. In the second case $y = \frac{2\pi}{3}$ or $\frac{4\pi}{3}$ and x could be anything from 0 to 2π. The largest sum is $\frac{4\pi}{3} + 2\pi = \frac{10\pi}{3} = \frac{20\pi}{6}$. Hence, the largest possible sum is $\boxed{\frac{23\pi}{6}}$.

T–7. Extend $\overline{QP}$ and $\overline{QT}$ so that they intersect circle O at V and S, respectively. By the Power of a Point Theorem, $AT \cdot TB = QT \cdot TS$. Since $QV = 20 \rightarrow QP = 16$ and $PT = 4$, then $QT^2 = 16^2 - 4^2$. Thus, $QT = 4\sqrt{15}$. Since $\Delta QVS \sim \Delta QPT$, then $\frac{QS}{QT} = \frac{QV}{QP} \rightarrow$

$\frac{QS}{4\sqrt{15}} = \frac{20}{16} \rightarrow QS = 5\sqrt{15} \rightarrow TS = \sqrt{15}$. Thus,

$QT \cdot TS = 4\sqrt{15} \cdot \sqrt{15} = 60 \rightarrow AT \cdot TB = \boxed{60}$.

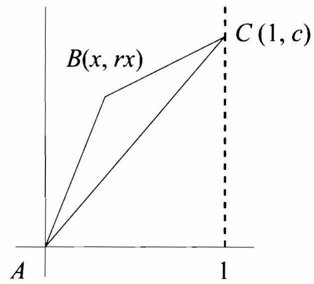


T–8. If $b = 0$, the parametric system is $x = at^3$, $y = at^3$ whose rectangular form is $y = x$ and that can't have two distinct y-intercepts. If $a = 0$, then we have $x = -bt$, $y = bt^2$ which turns into $y = \frac{x^2}{b}$ and that can't have two distinct y-intercepts. So, $a, b \neq 0$. The zeros for x occur at $t = 0, \pm\sqrt{\frac{b}{a}}$. Since $y(0) = 0$, for there to be two distinct y-intercepts, there are three cases to consider: i) $y\left(\sqrt{\frac{b}{a}}\right) = y\left(-\sqrt{\frac{b}{a}}\right)$, ii) $y\left(\sqrt{\frac{b}{a}}\right) = 0$, or iii) $y\left(-\sqrt{\frac{b}{a}}\right) = 0$. The first is impossible since $a, b \neq 0$. The second is impossible since $y\left(\sqrt{\frac{b}{a}}\right) = a \cdot \frac{b}{a}\sqrt{\frac{b}{a}} + b \cdot \frac{b}{a} \neq 0$ because $b \neq 0$. In the third case we have

$y\left(-\sqrt{\frac{b}{a}}\right) = a \cdot \frac{-b}{a}\sqrt{\frac{b}{a}} + b \cdot \frac{b}{a} = b\left(\frac{b}{a} - \sqrt{\frac{b}{a}}\right)$ and this equals 0 as long as $a = b$. Thus, the ordered pairs (a,a) yield the two distinct solutions (0, 0) and (0, $2a$) as long as a and b do not equal 0.

Thus $(a,b) = (1, 1), (2, 2), \ldots, (100, 100)$, giving $\boxed{100}$ answers.

T–9. Let the coordinates of C be $(1, c)$. Since $\frac{c - rx}{1 - x} = s$ then $c = s + (r - s)x$. Using determinants, the area of $\Delta ABC = \frac{1}{2}\begin{vmatrix} 1 & s + (r - s)x \\ x & rx \end{vmatrix} = \frac{1}{2}\left(x - x^2\right)(r - s)$. The maximum of $\frac{1}{2}(x - x^2)$ occurs at $x = \frac{1}{2}$ and equals $\frac{1}{8}$. Thus, $k = \boxed{\frac{1}{8}}$.



T–10. Since 10 contains the only zero, 10 must be in the 14th and 15th positions. Call positions 1 – 13 the front part of the palindrome and positions 16 – 29 the back part. If the number 11 lies in the front part in positions n and $n + 1$, then the number 1 must lie in the back part in position $29 - n$ and a number from 12 to 19 must fill positions $29 - (n - 1) = 30 - n$ and $29 - (n - 2) = 31 - n$. If the number 11 appears in the back part in positions n and $n + 1$, then the number 1 must appear in position $29 - n$ and a number drawn from 12 to 19 must appear in positions $29 - (n - 1) = 30 - n$ and $29 - (n - 2) = 31 - n$. Several conclusions can be drawn from this. First, the number 1 must be symmetric to one of the digits of the number 11. Second, if the number 11 lies in the front part, the largest value for $n + 1$ is 13, so the largest value for n is 12 and the smallest value for $29 - n$ is 17, implying that the number 1 cannot lie in the 16th position. Thus, a two-digit number must lie in positions 16 and 17. If the number 11 lies in the back part in positions n and $n + 1$, then the smallest value for n is 16 and the largest position for the number 1 would be the 13th. Thus, the two-digit number 1A must follow the two-digit number 10 and the one-digit number A must precede 10. Continuing in this fashion we see that our palindrome must have this form:

$$(I)(1H)(G)(1F)(E)(1D)(C)(1B)(A)(10)(1A)(B)(1C)(D)(1E)(F)(1G)(H)(1I)$$

with the letters $A, B, \ldots, I$ being any permutation of $1, 2, \ldots, 9$. Thus, there are $9! = \boxed{362880}$ permutations.