

NYCMT 2022-2023 HW#7

NYCMT

January 20 - February 3, 2022

These problems are due Friday, February 3rd. Please solve as many problems as you can, and write up solutions (**not just answers!**) to the ones you solve. Also write down any progress you make on problems you don't solve. Please write solutions for different questions on separate pages. Make sure to write your name on each page and page numbers per problem.

If you're not going to be present on February 3rd, you can scan your solutions and email them to jothman30@stuy.edu and jmoltz30@stuy.edu. If you will be there, just hand in your responses on paper. If you have any questions, just ask one of us on Discord or using one of the above emails.

Problems are not necessarily in difficulty order. Make sure to try them all!

Enjoy!

Problems

Problem 1. For Coco the Fruit's birthday, Stan the Kat gives him one brick weighing x pounds, two bricks weighing y pounds, and three bricks weighing z pounds, where x, y, z are positive integers of Coco the Fruit's choice. Coco the Fruit has a balance scale. By placing some combination of bricks on the scale (possibly on both sides), he wants to be able to balance any item of weight $1, 2, \dots, N$ pounds. What is the largest N for which Coco the Fruit can succeed?

Problem 2. Compute the largest n such that $n!$ can be written as the product of $n - 3$ consecutive integers.

Problem 3. In triangle $\triangle XYZ$, M_X , M_Y and M_Z are the midpoints of arcs YZ , ZX and XY in the circumcircle of $\triangle XYZ$. Show that the diagonals formed by opposite points in the hexagon formed by the intersection of triangles $\triangle XYZ$ and $\triangle M_X M_Y M_Z$ concur.

Problem 4. A fair 6-sided die is repeatedly rolled until an odd number appears. What is the probability that every even number appears at least once before the first occurrence of an odd number?

Problem 5. Let x be a positive real number. Find the maximum possible value of

$$\frac{x^2 + 2 - \sqrt{x^4 + 4}}{x}$$