

# NYCMT 2022-2023 HW#6

NYCMT

December 2 - December 9, 2022

These problems are due Friday, December 9th. Please solve as many problems as you can, and write up solutions (**not just answers!**) to the ones you solve. Also write down any progress you make on problems you don't solve. Please write solutions for different questions on separate pages. Make sure to write your name on each page and page numbers per problem.

If you're not going to be present on December 9th, you can scan your solutions and email them to [jothman30@stuy.edu](mailto:jothman30@stuy.edu) and [jmoltz30@stuy.edu](mailto:jmoltz30@stuy.edu). If you will be there, just hand in your responses on paper. If you have any questions, just ask one of us on Discord or using one of the above emails.

**Problems are not necessarily in difficulty order. Make sure to try them all!**

Enjoy!

## 1 Problems

**Problem 1.** Five students take a test in which any integer score from 0 to 100 is possible. What is the largest possible value of the difference of the mean of their scores and the median of their scores?

**Problem 2.** Find the smallest prime  $p \neq 11, 13$  such that  $p + p^{-1} \equiv 25 \pmod{143}$ . Here  $p^{-1}$  denotes the unique integer  $n$  less than 143 such that  $pn \equiv 1 \pmod{143}$ .

**Problem 3.** Let  $P$  and  $Q$  be points on line  $\ell$  with  $PQ = 12$ . Two circles,  $\omega$  and  $\Omega$ , are both tangent to  $\ell$  at  $P$  and are externally tangent to each other. A line through  $Q$  intersects  $\omega$  at  $A$  and  $B$ , with  $A$  between  $B$  and  $Q$ , such that  $AB = 10$ . Similarly, another line through  $Q$  intersects  $\Omega$  at  $C$  and  $D$ , with  $C$  between  $D$  and  $Q$ , such that  $CD = 7$ . Find the ratio  $AD/BC$ .

**Problem 4.** Find the number of fractions in the following list that are in simplest form.

$$\frac{1}{2014}, \frac{2}{2013}, \dots, \frac{1006}{1009}, \frac{1007}{1008}$$

**Problem 5.** Find the number of ways to choose 4 distinct integers from 1 to 13 such that at least 2 of them are consecutive.