

# NYCMT 2022-2023 HW#4

NYCMT

October 21 - October 28, 2022

These problems are due Friday, October 28th. Please solve as many problems as you can, and write up solutions (**not just answers!**) to the ones you solve. Also write down any progress you make on problems you don't solve. Please write solutions for different questions on separate pages. Make sure to write your name on each page and page numbers per problem.

If you're not going to be present on October 28th, you can scan your solutions and email them to [jothman30@stuy.edu](mailto:jothman30@stuy.edu) and [jmoltz30@stuy.edu](mailto:jmoltz30@stuy.edu). If you will be there, just hand in your responses on paper. If you have any questions, just ask one of us on Discord or using one of the above emails.

**Problems are not necessarily in difficulty order. Make sure to try them all!**

Enjoy!

## 1 Problems

**Problem 1.** Let  $a_1, a_2, a_3, a_4$ , and  $a_5$  be the first 5 terms of an arithmetic progression. Let  $b_1, b_2, b_3, b_4$ , and  $b_5$  be the first 5 terms of a geometric progression. Let  $c_n = a_n + b_n$ . If  $c_1 = c_2 = c_4 = 0$  and  $c_3 = 1$ , compute  $c_5$ .

**Problem 2.** Let  $p(x)$  be a monic polynomial such that  $p(x)$  is a factor of both  $x^4 + 6x^2 + 25$  and  $3x^4 + 4x^2 + 28x + 5$ . Compute  $p(1)$ .

**Problem 3.** How many sequences of nonnegative integers  $a_1, a_2, \dots, a_n$  (with  $n \geq 1$ ) are there that satisfy  $a_1 a_n > 0$ ,  $a_1 + a_2 + \dots + a_n = 10$ , and  $\prod_{i=1}^{n-1} (a_i + a_{i+1}) > 0$ ?

**Problem 4.** In parallelogram  $ABCD$  (with  $\angle ABC < 90^\circ$ ), the perpendicular from  $C$  to  $AB$  and the perpendicular from  $A$  to  $BC$  intersect at  $P$ . If  $PB = 96$  and  $PD = 580$ , compute  $AC$ .

**Problem 5.** How many integers  $N$  between 1 and 2022 (inclusive) satisfy that  $\frac{N^2+7}{N+4}$  is a fraction in simplest form?