NYCMT 2022-2023 HW#3

NYCMT

September 30 - October 7, 2022

These problems are due Friday, October 7th. Please solve as many problems as you can, and write up solutions (**not just answers!**) to the ones you solve. Also write down any progress you make on problems you don't solve. Please write solutions for different questions on separate pages. Make sure to write your name on each page and page numbers per problem.

If you're not going to be present on October 7th, you can scan your solutions and email them to jothman30@stuy.edu and jmoltz30@stuy.edu. If you will be there, just hand in your responses on paper. If you have any questions, just ask one of us on Discord or using one of the above emails.

Problems are not necessarily in difficulty order. Make sure to try them all!

Enjoy!

1 Problems

Problem 1. Let AB be the diameter of a semicircle ω centered at O. Let CD be a chord of ω, where AB and CD intersect at E which is outside of ω. Given that E is closer to A and C than it is to B and D, and that $EC = OB$ and $\angle DOB = 40^\circ$, compute $\angle DAB$.

Problem 2. John is learning how to shoot a basketball. To do this, he has purchased 4 indistinguishable magenta basketballs and 4 indistinguishable cyan basketballs to place on his basketball cart. His cart is very weird, so it has 4 rows and 4 columns; each slot can hold 1 basketball. Find, with proof, the number of ways there are for him to place his 8 basketballs in his basketball cart so that in each row and column, there is exactly 1 cyan basketball and exactly 1 magenta basketball.

Problem 3. Compute the sum of all x satisfying the equation $5^{2x+2} - 5^{x+3} - 5^x + 5 = 0$

Problem 4. 9^{4000} has 3817 digits, the leftmost of which is a 9. Given this information, find, with proof, the number of powers of 9 from 9^0 to 9^{4000} that begin with a 9.

Problem 5. Prove that there are no positive integer triples (a, b, c) such that $a^3 - b^3 = 2^c$.

Problem 6. The answers to the first 5 problems can be decoded to reveal the location of JoJo's treasure. Submit the treasure for full credit, no proof required.