

NYCMT 2022-2023 HW#2

NYCMT

September 23 - September 30, 2022

These problems are due Friday, September 30th. Please solve as many problems as you can, and write up solutions (**not just answers!**) to the ones you solve. Also write down any progress you make on problems you don't solve. Please write solutions for different questions on separate pages. Make sure to write your name on each page and page numbers per problem.

If you're not going to be present on September 30th, you can scan your solutions and email them to jothman30@stuy.edu and jmoltz30@stuy.edu. If you will be there, just hand in your responses on paper. If you have any questions, just ask one of us on Discord or using one of the above emails.

Problems are not necessarily in difficulty order. Make sure to try them all!

Enjoy!

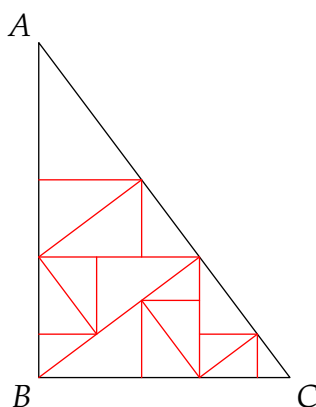
1 Problems

Problem 1. How many subsets S of $\{1, 2, \dots, 12\}$ satisfy $|S| \geq 2$ and the sum of the two largest elements of S is 15.

Problem 2. Find the interval of real values r such that

$$\lfloor r + \frac{1}{100} \rfloor + \lfloor r + \frac{2}{100} \rfloor + \lfloor r + \frac{3}{100} \rfloor + \dots + \lfloor r + \frac{70}{100} \rfloor + \lfloor r + \frac{71}{100} \rfloor = 619$$

Problem 3. Consider right triangle ABC with $AB = 4$, $BC = 3$, $CA = 5$. Construct the altitude in triangle ABC to the hypotenuse. This splits ABC into 2 right triangles. Construct the altitude to the hypotenuse in these 2 triangles creating 4 triangles. Repeating this process forever a fractal made up of the altitudes is constructed. Does the fractal have finite length? If yes compute it. If not, justify your answer.



Fourth level of the fractal

Problem 4. Rectangle $ABCD$ has $AB = 22$ and $BC = 10$. Assume that there exists some $\triangle PQR$ that has orthocenter A , circumcenter B , C as the midpoint of QR , and D on QR . Find, with proof, the length of QR .

Problem 5. Given that n has 10 factors the sum of which is 248, compute n .