NYCMT 2022-2023 HW#1

NYCMT

September 16 - September 23, 2022

These problems are due Friday, September 23rd. Please solve as many problems as you can, and write up solutions (**not just answers!**) to the ones you solve. Also write down any progress you make on problems you don't solve. Please write solutions for different questions on separate pages. Make sure to write your name on each page and page numbers per problem.

If you're not going to be present on September 23rd, you can scan your solutions and email them to jothman30@stuy.edu and jmoltz30@stuy.edu. If you will be there, just hand in your responses on paper. If you have any questions, just ask one of us on Discord or using one of the above emails.

Problems are not necessarily in difficulty order. Make sure to try them all!

Enjoy!

1 Problems

Problem 1. The equation $742586 + 829430 = 1212016$ is incorrect. However, we can change one digit d in the equation to another e to create a true equation. Find, with proof, the ordered pair (d, e).

Problem 2. Three circles of radius 3 have centers $(2, 22)$, $(5, 6)$, and $(7, 14)$, respectively. A line passing through $(5, 6)$ is drawn such that the sum of the areas of the parts of the circles on each side of the line are equal. What is the y-intercept of this line?

Problem 3. Let $P(x) = x^6 + 8x^5 + 12x^4 + 3x^3 + 7x^2 + 14x + 2$ be a polynomial with roots $r_1, r_2, \cdots r_6$. Compute

$$\sum_{1 \le i < j \le 6} \frac{(r_i + r_j)^2}{r_i r_j}.$$

Problem 4. Compute the number of triples of positive integers (a, b, c) such that $18!|a|b|c|21!$.

Problem 5. Define the Fibonacci sequence as follows, $F_1 = 1$, $F_2 = 1$ and $F_n = F_{n-1} + F_{n-2}$ for all $n > 2$. Find all solutions j for which

(a) $F_j = j$

(b) $F_j = j^2$

Problem 6. Prove or disprove: every positive integer at least 6 can be written as the sum of 3 primes.