v_p and LTE

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December 2020

1 Problems

1. Let n be the least positive integer for which $149^n - 2^n$ is divisible by $3^3 \cdot 5^5 \cdot 7^7$. Find the number of positive integer divisors of n.

2. Find the smallest n such that 2013^n ends in 001 (PUMAC 2013 2)

3. For a given k, find all n such that 5^k divides $2^n - 1$

4. $a^{a-1} - 1$ is never squarefree for $a > 2$

5. Let $p_1 = 2012$ and $p_n = 2012^{p_{n-1}}$ for $n > 1$. Find the largest integer k such that $p_{2012} - p_{2011}$ is divisible by 2011^k. (PUMAC 2012 6)

6. (ISL 1991) Find the largest k such that $1991^k | 1990^{1991^{1992}} + 1992^{1991^{1990}}$

7. Show that if n is square free and if x, y are relatively prime, $\dfrac{x^n + y^n}{(x+y)^3}$ is never an integer

8. Let $a, b > 1$ be positive integers and suppose $(a^n - 1)(b^n - 1)$ is a square for all positive integers n. If p is prime then the order of a mod p is the same as the order of b mod p.

   The order of a mod p is the smallest k such that $a^k \equiv 1 \mod p$

9. Determine all n such that $\dfrac{2^n + 1}{n^2}$ is an integer (IMO 1990 3) Please don't get this because I really tried for you to not be able to run out of material.