



# The Mandelbrot Competition

## Round Three Test

Time Limit:  
40 minutes

Name: \_\_\_\_\_

1. One can fill in the squares with the digits from 1 to 7 so that each digit appears exactly once and each pair of adjacent digits, viewed as a two-digit number, is divisible by 7 or 9. What is the resulting seven-digit number?

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①

2. Four points  $A, B, C, D$  in the plane are situated so that  $AB = 4$ ,  $BC = 5$ ,  $CD = 6$  and  $DA = 8$ . What is the minimum possible value of distance  $AC$ ?

①

3. Oliver rolls a fair die and flips a fair coin. If the coin comes up heads, he multiplies the number on the die by 2. Otherwise, he multiplies the number on the die by 3. What is the probability that the result is divisible by 6?

②

4. We say that a positive integer  $m$  is *fortunate* if there are three distinct primes larger than 4 such that dividing  $m$  by each of these three primes gives a remainder of 4. Determine the smallest multiple of 3 that is fortunate.

②

5. Let  $\triangle ABC$  be an isosceles triangle with  $AB = BC$  and  $m\angle BAC = 40^\circ$ . Suppose the angle bisector of  $\angle BAC$  intersects the circumcircle of  $\triangle ABC$  at  $D$ , while the angle bisector of  $\angle ADC$  intersects this circumcircle at  $E$ . Compute  $m\angle DAE$ , in degrees.

②

6. Find the exact value (in terms of  $\pi$ ) of the expression shown at right, if there are 2017 minus signs appearing in this continued fraction.

$$2 - \frac{2}{2 - \frac{2}{2 - \frac{2}{\ddots 2 - \frac{2}{2 - \pi}}}}$$

③

7. Consider the function  $f(x) = x^2 + 20x + 17$ . For what value of  $a$  are both the solutions to  $f(x) = x$  also solutions to  $a - f(a - f(x)) = x$ ?

③

SCORE: