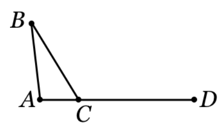
Answer Key			
1.	7214563	4.	459
2.	2	5.	70
3.	5/12	6.	$\frac{2}{2-\pi}$
		7.	-19

1. Finding the solution to this digit puzzle is mainly an exercise in organized trial and error. For instance, the digit 7 must be followed by a 2, unless it is the final digit. Similarly the 1 must be followed by the 4, if anything. Continuing in this manner one discovers the string of digits **7214563** before long. To our delight, this is the unique answer.

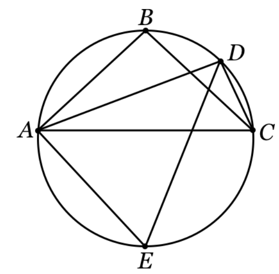
2. Because $AD = 8$ while $CD = 6$, it is impossible for point A to be any closer than **2** units to point C. Think of placing D at the center of the diagram and letting C circle around 6 units away, while A circles around 8 units away. Then A and C will be nearest when they both line up on one side of D, as shown in the diagram.



3. We can obtain a multiple of 6 in two distinct ways: either the coin comes up heads and the die shows a 3 or 6, or else the coin comes up tails and the die shows a 2, 4 or 6. The probability of the former is $(\frac{1}{2})(\frac{1}{3})$ while the probability of the latter is $(\frac{1}{2})(\frac{1}{2})$. Hence the overall probability is $\frac{1}{6} + \frac{1}{4} = \mathbf{\frac{5}{12}}$.

4. To build a fortunate number select primes $p, q, r \geq 5$, then add 4 to some multiple of pqr. Clearly this number leaves a remainder of 4 when divided by either p, q or r. For instance, the smallest fortunate number is $(5)(7)(11)+4 = 389$. Unfortunately, it is not a multiple of 3. Another fortunate number is $2(5)(7)(11) + 4 = 774$, which is a multiple of 3. Unfortunately, it is not the smallest such number! We can do better using another triplet of primes; namely $(5)(7)(13)+4 = \mathbf{459}$, giving the smallest fortunate multiple of 3.

5. The diagram at right summarizes the situation. The staple tool in our solution will be the Intercepted Arc Theorem, which states that the measure of an inscribed angle is equal to half the measure of its intercepted arc. For instance, $m\angle BAC = 40°$, therefore $m\widehat{BC} = 80°$. In the same manner, angle bisector $\overline{AD}$ creates two 20° angles and two 40° arcs. Using this result in reverse, we find that $m\angle BCD = 20°$ as well, since it intercepts $\widehat{BD}$, and $m\widehat{BD} = 40°$.



Focusing on $\triangle ACD$, we find $m\angle ACD = 60°$, which means that $m\angle ADC = 100°$ (we know $m\angle DAC = 20°$), giving $m\widehat{AC} = 200°$. As before, the angle bisector $\overline{DE}$ splits both the angle and the intercepted arc in half, creating a pair of 50° angles and a pair of 100° arcs. Finally, we see that $\angle DAE$ intercepts a total of 140° worth of arc, which provides our answer of $m\angle DAE = \frac{1}{2}(140°) = \mathbf{70°}$.

6. Our only hope, it would seem, is to work our way up from the bottom and hope to discover some sort of pattern. Using one minus sign we have $\frac{2}{2-\pi}$. Incorporating the second minus sign yields

$$\frac{2}{2-\frac{2}{2-\pi}} = \frac{2}{\frac{2-2\pi}{2-\pi}} = \frac{2-\pi}{1-\pi}.$$

Moving on to the third minus sign produces

$$\frac{2}{2-\frac{2-\pi}{1-\pi}} = \frac{2}{\frac{-\pi}{1-\pi}} = \frac{2\pi-2}{\pi},$$

which appears to be neither better nor worse than before. The light at the end of the tunnel becomes visible with the fourth minus sign,

$$\frac{2}{2-\frac{2\pi-2}{\pi}} = \frac{2}{\frac{2}{\pi}} = \pi.$$

Hence the fifth minus sign brings us back to $\frac{2}{2-\pi}$, and the whole pattern repeats with period four. In particular, the expression will return to $\frac{2}{2-\pi}$ after the 2017th minus sign as well.

7. The equation $f(x) = x$ can be rearranged to give $x^2 + 19x + 17 = 0$, which is a quadratic having two real roots r and s with sum $r+s = -19$ and product $rs = 17$. Since r is required to be a root of the second equation as well, we must have

$$a - f(a - f(r)) = r \quad \Longrightarrow \quad f(a-r) = a - r,$$

where we used the fact that $f(r) = r$ since r is a solution to $f(x) = x$. In other words, $a - r$ must also be a root of $f(x) = x$, which means that either $a - r = r$ or $a - r = s$. In the first case we have $a = 2r$, while the second case leads to $a = r + s = -19$.

Using similar reasoning, the fact that s must also be a root of the second equation implies that either $a = 2s$ or $a = -19$. The only value of a for which both r and s are solutions to the second equation is $a = \mathbf{-19}$.

Problem Credits

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⋆ Regional Level ⋆

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