⋆ Regional Level ⋆

October 2017

The Mandelbrot Competition

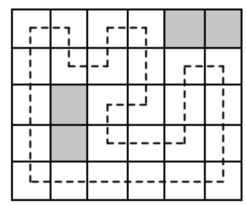
Round One Test

Time Limit: 40 minutes

Name: ______________________________

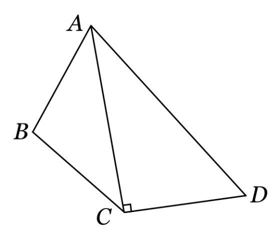
1. There is a positive integer with the property that adding 3 to this number, taking the square root of the result, adding 3 again, then dividing the result by 3 yields a grand total of 3. Find this positive integer. 1

2. What is the maximum number of 1×2 dominoes one can place on a 5×6 grid such that a connected loop can be drawn through the remaining squares, if no two dominoes overlap or occupy adjacent squares? (The dotted loop may only move horizontally or vertically, not diagonally.) 1



3. Let $f(x) = ax^2 + bx + c$ be a quadratic with positive integer coefficients. If $f(1) = 21$ and $f(10) = 201$, then what is $f(100)$? 2

4. Let $\triangle ABC$ be an isosceles triangle with $AB = BC$ and $m\angle ABC = 100°$. A point D is selected outside of $\triangle ABC$ such that $\angle DCA$ is a right angle and $DC = BC$. Compute $m\angle DBC$ in degrees. 2



5. For how many positive integers b does the base b expansion of π start out as $3.1\ldots$? (Note $b = 10$ works, since in base 10 we have $\pi = 3.14159\ldots$.) 2

6. Let us say that a decade is *primeval* if it contains four prime numbers. For instance, the decade from 1480 to 1490 was primeval, since 1481, 1483, 1487 and 1489 are all primes. Let p_1, p_2, p_3 and p_4 be the primes (in order) in the next primeval decade after 2020. Compute the value of $p_2p_3 - p_1p_4$. 3

7. A certain collection of numbered index cards includes one card with a 1 written on it, two cards with a 2, and so forth up to n cards showing an n, for some positive integer n. Determine n, if it is the case that drawing a card at random from this collection results in a value of 2017, on average. 3

SCORE: