Team Round

1. Compute $\dfrac{\sqrt{\sqrt{5}+2}-\sqrt{\sqrt{5}-2}}{\sqrt{\sqrt{5}-1}}$.

2. Let k be a positive integer such that the product $2015k$ has exactly 18 divisors. Compute the sum of all possible values of k.

3. Trapezoid $ABCD$ has $\overline{AB} \parallel \overline{CD}$ and $AB = BC$. A circle with center O on $\overline{CD}$ is tangent to $\overline{DA}$, $\overline{AB}$, and $\overline{BC}$. If $m\angle BCD = 68^\circ$, compute $m\angle CDA$.

4. If the sum of the cubes of the roots of $x^2 - bx + 10 = 0$ is $6b$, compute the greatest possible value of b.

5. Compute the maximum value of $\log x + \log y + \log z$ if x, y, and z are positive real numbers that satisfy $x + 4y + 16z = 120$.

6. A broken automatic card shuffler can only shuffle decks of five cards. Furthermore, it can only perform two types of shuffles. The first type of shuffle only switches the top two cards (leaving the order of the bottom cards unchanged), and the second type of shuffle reverses the order of the bottom four cards (leaving the top card on top). Five cards are numbered with five different numbers and arranged in a random order, with all permutations equally likely. What is the probability that the shuffler cannot put the cards in increasing order, from top to bottom, regardless of the number and types of shuffles it performs?

7. There are 12 adjacent parking spaces in a parking lot and 8 of them are occupied. A large truck arrives, needing 2 adjacent unoccupied spaces to park. Compute the probability that it will be able to park. (Each arrangement of cars is equally likely.)

8. If $x + y + z = 7$, $xy + yz + zx = 8$, and $xyz = 2$, compute the maximum value of x.

9. The three-digit prime number p is written in base 2 as p_2 and in base 5 as p_5, and the two representations share the same last two digits. If the ratio of the number of digits in p_2 to the number of digits in p_5 is 5 to 2, find all possible values of p.

10. The parabola $y = 3x^2 + 5x - 9$ has focus F. A line passes through F and intersects the parabola at two points, $P_1 = (-4, 19)$ and P_2. Two tangent lines to the parabola are drawn through P_1 and P_2, intersecting at Q. Compute $m\angle P_1 Q P_2$.

Answers

Team Round

1. $\sqrt{2}$
2. 623
3. 44°
4. 6
5. 3
6. $\frac{9}{10}$
7. $\frac{41}{55}$
8. $3+\sqrt{7}$
9. 601
10. 90°