NYC Math Team Practice Session — Stan Kats
Number Theory — 9/25/15

1. How many PRTs are there with hypotenuse less than or equal to 25? (How many are Primitive?)

2. Show that the only PT (x, y, z) where x, y and z are consecutive integers is $(3,4,5)$.

3. Find "Almost Pythagorean Triples" (a, b, c) such that $a^2 + b^2 = c^2 + 1$.

4. Find some P(rimitive) PRTs such that a leg and the hypotenuse are consecutive integers.

5. Find a Quadrilateral whose sides and whose area are integers.

6. Find a solution in positive integers of $\frac{1}{x^2} + \frac{1}{y^2} = \frac{1}{z^2}$.

The next three problems may be slightly more demanding

7. Find solutions in positive integers of $w^2 + x^2 + y^2 = z^2$.

8. Find solutions in positive integers of $x^2 + 2y^2 = z^2$. **

9. Find solutions in positive integers of $x^2 + 3y^2 = z^2$. **

** Investigate triples of the form $x = \frac{m^2 - pn^2}{2},\ y = mn,\ z = \frac{m^2 + pn^2}{2}$ where $m > n\sqrt{p}$, p is a prime, and m, n are suitably chosen. Also investigate the relative primality of x, y, and z for these triples.