New York City Math Team
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Definition:
A polynomial equation of degree n in one variable is an equation of the form
$a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0 = 0 \quad (a_n \neq 0)$
where a's represent constants and the exponents of the variable are nonnegative integers.

- The coefficient a_n is called the leading coefficient.
- If the leading coefficient is 1, the polynomial is *monic*.

Method of Undetermined Coefficients

1) Polynomial $P(x) = ax^5 - 3x^3 + bx^2 + 17x + c$ and $Q(x) = 4x^5 + dx^4 + ex^3 - 11x^2 + fx - 9$ are equal. Find the coefficients a, b, c, d, e, and f.
2) $P(x) = x^2 + 3x - 7$ is a factor of $Q(x) = 3x^4 + 11x^3 - 19x^2 - 26x + 28$. Find the quotient $Q(x)/P(x)$ without dividing the two polynomials.
3) Find the quotient and the remainder when $4x^5 - 3x^3 + 2x^2 - x + 8$ is divided by $3x^3 + 7x - 5$, using the method of undetermined coefficients.
4) Let $f(x)$ and $g(x)$ be two quadratic polynomials such that $f(x) = 2x^2 - 3x + 5$ and $f\big(g(x)\big) = 18x^4 - 21x^2 + 10$. Find $g(x)$. (AoPS 20.15)
5) Suppose p is a polynomial such that $p(1) = 3$ and $p(x) = p(2x) - 6x^2 + 3x$. Find $p(x)$. (AoPS 20.16)
6) Find the square root of $3 + 18i$ and write the answer in the standard form.
7) The equation $4x^4 + px^3 - 23x^2 + qx + 36 = 0$ has two double roots. Find the values of p and q.
8) Find all roots of the equation $(x^2 + 4x - 2)^2 + 4(x^2 + 4x - 2) - 5 = 0$.
9) Solve the equation $(x^2 + 4x - 2)^2 + 4(x^2 + 4x - 2) - 2 = x$, using the method of undetermined coefficients.

10) Real roots x_1 and x_2 of the equation $x^5 - 55x + 21 = 0$ satisfy $x_1 x_2 = 1$. Find x_1 and x_2. (Bosnia and Herzegovina)

11) Solve for all complex numbers z such that $z^4 + 4z^2 + 6 = z$. (HMMT)

12) If $x + y + z = 13$ and $7x + 3y = -9$, find the value of $5x + y - 2z$. (Tkachuk)

13) Find the square root of $20 - 16i$ and write the answer in the standard form.

14) Find all the root of the equation $x^4 - 15x^2 + 12x + 5 = 0$.