Combinatorics Problems
NYC Math Team Practice
December 11, 2015

1. a. Find the number of solutions to $x + y + z = 8$ where x, y and z are positive integers.

   b. Find the number of solutions to $x + y + z = 9$ where x, y and z are positive integers.

   c. Find the number of solutions to $w + x + y + z = 8$ where w, x, y and z are positive integers.

   d. Find the number of solutions to $w + x + y + z = 9$ where w, x, y and z are positive integers.

2. a. Find the number of solutions to $x + y = 4$ where x and y are non-negative integers. That is integers greater than or equal to 0.

   b. Find the number of solutions to $x + y + z = 4$ where x, y and z are non-negative integers.

   c. Find the number of solutions to $x + y = 5$ where x and y are non-negative integers. That is integers greater than or equal to 0.

   d. Find the number of solutions to $x + y + z = 5$ where x, y and z are non-negative integers.

   e. Find the number of solutions to $w + x + y + z = 5$ where w, x, y and z are non-negative integers.

3. a. Find the number of solutions to $x + y = 8$ where x and y are integers with $x \geq 1$ and $y \geq 2$.

   b. Find the number of solutions to $x + y + z = 8$ where x, y and z are integers with $x \geq 1$, $y \geq 2$ and $z \geq 3$.

4. Find the number of solutions to $x + y + z = 8$ with x, y and z integers with $0 \le x < y < z$.

5. a. If the expression $(x + y + z)^{10}$ is simplified by expanding it and combining like terms, how many terms are in the simplified expression?

b. If the expression $(x + y + z)^{10} + (x - y - z)^{10}$ is simplified by expanding it and combining like terms, how many terms are in the simplified expression?
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