Combinatorics

1. How many ways can you arrange a king, a queen, and three pawns on five consecutive squares on a chessboard?
2. How many ways can you place a king, a queen, and three pawns on eight squares of the first rank of a chessboard?
3. How many three-letter strings can be created with the letters of the word PENCIL without repetitions? with repetitions?
4. How many four-letter strings can be created with a ten letter alphabet with repitions? without repetitions?
5. How many three-digit numbers can be formed using the number 2,3,4,5,6, and 8 if:
   (a) repetitions of digits are allowed?
   (b) repetitions are not allowed?
   (c) the number is to be odd and repetitions are not allowed?
   (d) the number is to be even and repetitions are not allowed?
   (e) the number is to be a multiple of 5 and repetitions are not allowed?
   (f) the number must contain the digit 5 and repetitions are not allowed?
   (g) the number must contain the digit 5 and repetitions are allowed?
6. How many ways can five people be seated on a straight bench?
7. There are 20 people at a bus stop with a bench that seats only five people. How many ways can five people be seated on the bench?
8. There are 20 people at a bus stop with a bench that seats only five people. How many different sets of five people can be seated on the bench?
9. How many different ways are there to arrange seven E's and five N's in a row.
10. Find the number of ordered triples (x, y, z), where x, y, and z are distinct integers between 1 and 10, inclusive.
11. Find the number of sets $\{x, y, z\}$, where x, y, and z are distinct integers between 1 and 10, inclusive.
12. Find the number of five-letter strings in which A appears at least once.
13. How many four-digit integers are there that contain exactly one 8?
14. How many integers from 1 to 9999 have distinct digits?
15. There are 10 people at a meeting. Each person will shake hands with every other person. How many handshakes will there be.
16. Evaluate:
    a. ${}_{10}P_3$ b. ${}_{10}P_7$ c. $\frac{{}_9P_4}{{}_4P_4}$

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17. A class has 17 girls and 14 boys.
    (a) How many different committees of five students can be formed?
    (b) How many different committees of five students can be formed if the committee must consist of two boys and three girls.
    (c) How many different committees of five students can be formed if the committee must have at least one boy and one girl?

18. Find the number of:
    (a) four-person committees that can be chosen from a class of 11.
    (b) b. seven-person committees that can be chosen from a class of 11.

19. There are 9 different books on a shelf; 4 are red and 5 are green. Find the number of possible arrangements for the books if:
    (a) the red books must be together and the green books must be together.
    (b) the red books must be together.
    (c) the colors must alternate.

20. How many different ways can you walk from 9^{th} Avenue and 14^{th} Street to 3^{rd} Avenue and 54^{th} Street if you can only walk north and east?

21. Find the number of diagonals in a dodecagon. In and n-gon.

22. Find the coefficient of the x^4y^8 term in the expansion of $(x+y)^{12}$.

23. Find the coefficient of the x^3y^5 term in the expansion of $(x+2y)^8$.

24. Evaluate:
    a. ${}_9C_5$ b. ${}_9C_4$ c. ${}_{20}C_{20}$ d. ${}_{20}C_1$

25. Consider a tournament in which each of n teams plays against every other team and each team wins at least once. Show that there are at least two teams having the same number of wins.

26. If 5 points are chosen at random in the interior of an equilateral triangle each side of which is 2 units long, show that at least 1 pair of points has a separation of less than 1 unit.

27. Show that any set of n integers has a subset such that the sum of the integers in the subset is divisible by n.

28. How many ordered triples in non-negative integers satisfy the equation $x+y+z=16$.

29. How many ordered triples in positive integers satisfy the equation $x+y+z=16$.

30. What is the sum of the enttries in the n^{th} row of Pascal's Triangle?