1. If
$$S = 1 + \cfrac{1}{2 + \cfrac{1}{3 + \frac{1}{4}}}$$
compute the value of S.

2. If
$$\frac{58}{15} = 3 + \cfrac{1}{1 + \cfrac{1}{6 + \frac{1}{x}}}$$
compute the value of x.

3. Investigate the following sequence of fractions:
   (a) $\frac{A_1}{B_1} = 1 + \frac{1}{1}$
   (b) $\frac{A_2}{B_2} = 1 + \cfrac{1}{1 + \frac{1}{1}}$
   (c) $\frac{A_3}{B_3} = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \frac{1}{1}}}$
   (d) $\frac{A_4}{B_4} = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \frac{1}{1}}}}$
   (e) Consider the sequence $1, 1, A_1, A_2, A_3, A_4, \ldots$ Predict the next three terms.

4. If $S = 2 + \frac{1}{6+}\frac{1}{1+}\frac{1}{1+}\frac{1}{11+}\frac{1}{2}$, compute S.

5. Compute $[3; 3, 1, 2, 2, 1, 9]$.

6. Compute $[0; 3, 3, 1, 2, 2, 1, 9]$

7. Express each of the following fractions as continued fractions in two ways:
   (a) $\frac{159}{86}$ (b) $\frac{79}{13}$ (c) $\frac{251}{802}$

8. Compute S if
$$S = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \ddots}}}$$

9. Compute S if $S = [2; \overline{2}]$

10. Compute S if $S = [1; \overline{2, 1}]$

11. Compute S if $S = [3; \overline{1, 4}]$

12. Compute S if $S = 1 + \frac{1}{2+}\frac{1}{3+}\frac{1}{2+}\frac{1}{3+}\ldots$

13. Compute $[3; \overline{1, 2, 3}]$.

14. Find a continued fraction expansion for the following:
   (a) $\sqrt{5}$
   (b) $\sqrt{7}$
   (c) $\sqrt{11}$
   (d) $\sqrt{19}$

15. Find a continued fraction expansion for 3.14159.
16. Find a continued fraction expansion for 2.71828.
17. Using the continued fraction expansions from 14 above, find the four convergents for $\sqrt{5}$ and $\sqrt{7}$.
18. Using the continued fraction expansions from 14 above, find the five convergents for $\sqrt{19}$.
19. Using the continued fraction expansions from 15 above, find the four convergents for 3.14159.
20. Complete the following table:
21. Use the continued fraction expansions found in 14 to find positive integer solutions to the following equations:
    (a) $x^2 - 5y^2 = 1$
    (b) $x^2 - 7y^2 = 1$
    (c) $x^2 - 19y^2 = 1$
22. Find three solutions in positive integers to $x^2 - 11y^2 = 1$.