New York City Math Team

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Methods of Proofs

1) Prove that the sum of two odd numbers is an even number.
2) Prove that the difference between any two consecutive perfect squares is an odd number.
3) Prove that for all natural numbers n the expression $\mathrm{n}(2\mathrm{n}-1)(2\mathrm{n}+1)$ is divisible by 3.
4) Prove that the statements $A \rightarrow B$ (if A then B) and $\sim B \rightarrow \sim A$ (if not A then not B) are equivalent.
5) Prove that there are infinitely many prime numbers.
6) Prove that the sum of three consecutive squares is not divisible by 3.
7) Prove that $19^{2016}-1$ is divisible by 5.
8) Prove that any prime number greater than 3 can be expressed as $6n-1$ or $6n+1$.
9) Six numbers 1, 2, 3, 4, 5, and 6 are written on a sheet of paper. At each move, you can add 1 to any two of the numbers on the list. Prove that it is impossible to get all the numbers on the list equal regardless of the number of moves.
10) Four people are sitting in a row. At each move, you can ask any two people switch their places. However, the remaining two people must switch their places as well. Prove that there exists a pair that cannot end up sitting next to one another regardless of the number of the moves.
11) Prove that if a and b are the legs and c is the hypotenuse of a right triangle, then $r=\frac{1}{2}(a+b-c)$, where r is the inradius.
12) Prove that 4^7+7^{16} is a composite number.
13) Prove that for all integer values of n, the expression $\frac{n^3}{6}+\frac{3n^2}{2}+\frac{13n}{3}+4$ is an integer.
14) Prove that if $a+b+c=0$, then $a^3+b^3+c^3=3abc$.

15) Five different points are chosen on the surface of a sphere. Prove that there exists a closed hemisphere (half a sphere that includes boundary) that contains four of the points.

16) Prove the inequality $\frac{1}{2} \cdot \frac{3}{4} \cdot \frac{5}{6} \cdot \ldots \cdot \frac{2015}{2016} < \frac{1}{44}$

17) Use the diagram below to prove that

$$1^3 + 2^3 + 3^3 + \ \ldots \ + n^3 = (1 + 2 + 3 + \ \ldots \ + n)^2$$