Bijections

Name:______________________________ Date:________________

Simple Ideas

1. A quiz consists of 10 true/false questions. If exactly seven of the answers are true, how many possible answer keys are there?

2. In a regular octagon, all diagonals are drawn. Compute how many interior points lie on two or more diagonals.

3. The number of different 10-letter "words" that can be made from the letters of the word REASSESSES is the same as the number of different x-letter "words" that can be made from the letters of the word REDUCTIONS. Compute x. [Note: The term "word" refers to any arrangement of letters.]

4. Poker chips come in three colors, red, white, and blue. Compute the number of different combinations there are of ten poker chips.

5. The faces of a cube are each painted in one of two colors: red or blue. In how many different ways can the six faces of the cube be painted? Two assignments of colors are not considered different if one can be obtained from the other with a rotation.

6. How many positive integers less than 100 are divisible by 2 or divisible by 3 (or both)?

Add a Twist

7. Compute the number of integers between 1000 and 5000 that are perfect squares.

8. Let $\mathcal{S}$ be the set $\{1, 2, 3, \ldots, 10\}$. Let n be the number of sets of two non-empty disjoint subsets of $\mathcal{S}$. (Disjoint sets are defined as sets that have no common elements.) Compute n.

9. Three fair dice are rolled and their sum is 6. Compute the probability that all three dice show a 2.

10. If x is an integer and $300 \leq x \leq 600$, compute the number of possible values of x such that it consists of 3 digits in ascending order.

11. Compute the number of terms when the expansion of $(a + b + c + d)^{17}$ is expressed in simplest form.

12. A row of beads consists of 10 red beads and 5 blue beads. Compute the number of possible arrangements of the beads that have no two blue beads adjacent.

13. Let $W = (0, 0)$, $A = (7, 0)$, $S = (7, 1)$, and $H = (0, 1)$. Compute the number of ways to tile rectangle $WASH$ with triangles of area $\frac{1}{2}$ and vertices at lattice points on the boundary of $WASH$.

14. Find the number of five-digit positive integers such that the sum of the digits is 17.

15. Let T be TNYWR. Compute the number of ordered T-tuples $(d_1, d_2, d_3, \ldots, d_T)$ of divisors of 24 such that $d_1 d_2 d_3 \cdots d_T$ is not a multiple of 24. (This is one part of a relay problem. Express your answer in terms of T.)

For Next Time

16. Compute the number of ordered triples (x, y, z) of positive integers such that $x + y + z \leq 6$.

17. Compute the number of three-digit integers that have the property that each digit except the last is less than the digit to its right.

18. Alice's coin purse contains 6 pennies, 8 nickels, 3 dimes, and 9 quarters. Bob borrows 10 of those coins, probably with Alice's permission. Determine the number of combinations of 10 coins Bob could have taken.

19. Prove that the number of positive integers less than n that are relatively prime to n is equal to $n\left(1-\frac{1}{p_1}\right)\left(1-\frac{1}{p_2}\right)\left(1-\frac{1}{p_3}\right)\cdots\left(1-\frac{1}{p_m}\right)$, where $p_1, p_2, p_3, \ldots, p_m$ are the prime factors of n.