

Combinatorics Problems
NYC Math Team Practice
Fall 2016

1.
 - a. Find the number of solutions to $x + y + z = 8$ where x, y and z are positive integers.
 - b. Find the number of solutions to $x + y + z = 9$ where x, y and z are positive integers.
 - c. Find the number of solutions to $w + x + y + z = 8$ where w, x, y and z are positive integers.
 - d. Find the number of solutions to $w + x + y + z = 9$ where w, x, y and z are positive integers.
2.
 - a. Find the number of solutions to $x + y = 4$ where x and y are non-negative integers. That is integers greater than or equal to 0.
 - b. Find the number of solutions to $x + y + z = 4$ where x, y and z are non-negative integers.
 - c. Find the number of solutions to $x + y = 5$ where x and y are non-negative integers. That is integers greater than or equal to 0.
 - d. Find the number of solutions to $x + y + z = 5$ where x, y and z are non-negative integers.
 - e. Find the number of solutions to $w + x + y + z = 5$ where w, x, y and z are non-negative integers.
3.
 - a. Find the number of solutions to $x + y = 8$ where x and y are integers with $x \geq 1$ and $y \geq 2$.
 - b. Find the number of solutions to $x + y + z = 8$ where x, y and z are integers with $x \geq 1$, $y \geq 2$ and $z \geq 3$.

4. Find the number of solutions to $x + y + z = 8$ with x, y and z integers with $0 \leq x < y < z$.
5. a. If the expression $(x + y + z)^{10}$ is simplified by expanding it and combining like terms, how many terms are in the simplified expression?
- b. If the expression $(x + y + z)^{10} + (x - y - z)^{10}$ is simplified by expanding it and combining like terms, how many terms are in the simplified expression?
[2006 AMC 12 A #24]
6. The numbers 1447, 1005 and 1231 are each 4-digit numbers beginning with 1 that have exactly two identical digits. How many numbers have this property? [AIME 1983 #10]
7. An object starts at $(0, 0)$ and moves in steps of length one either up or right.
- a. How many paths are there from $(0, 0)$ to $(4, 3)$ of length 7?
- b. How many paths are there from $(0, 0)$ to $(3, 3)$ of length 6?
- c. What is the probability that a path to $(4, 3)$ passes through $(3, 3)$? [1982 AHSME #25]
- d. If the object **cannot** pass through $(2, 1)$, how many possible paths are there from $(0, 0)$ to $(4, 3)$ of length 7?
8. Objects A and B move simultaneously in the coordinate plane via a sequence of steps of length one. Object A starts at $(0, 0)$ and each of its steps is either right or up, both equally likely. Object B starts at $(5, 7)$ and each of its steps is either left or down, both equally likely. What is the probability that the objects meet? [2003 AMC 12A #22]
9. Alice, Bob, Carl, Dan and Edna are to be seated in a row. How many possible seatings are there if Carl and Dan refuse to sit next to each other? [NYCIML SP 1980 #1]
10. If Alice, Bob, Carl, Dan and Edna are instead seated at a circular table, how many possible seatings are there if Carl and Dan refuse to sit next to each other?
11. A room is filled with Baltimore Ravens fans. If each person shakes hands with each other person, and there are 870 handshakes in all, how many people are in the room?
[NYCIML SP 1980 #7]