1. Compute the number of paths from (1,0) to (4,6).

2. A fair die is rolled four times. The probability that each of the final three rolls is at least as large as the roll preceding it may be expressed in the form $\frac{m}{n}$ where m and n are relatively prime positive integers. (AIME)

3. Compute the number of subsets of the set $\{a_1, a_2, a_3, \ldots, a_n\}$

4. Let set $S = \{1,2,3,4,5,6\}$, and let set T be the set of all subsets of S (including the empty set and S itself). Let t_1, t_2, t_3 be elements of T, not necessarily distinct. The ordered triple (t_1, t_2, t_3) is called *satisfactory* if either

   (a) both $t_1 \subseteq t_3$ and $t_2 \subseteq t_3$, or
   (b) $t_3 \subseteq t_1$ and $t_3 \subseteq t_2$.

   Compute the number of *satisfactory* ordered triples (t_1, t_2, t_3). (ARML Tiebreaker)

5. (a) A rectangular grid is obtained by tiling a 4-by-6-rectangle $ABCD$ with squares of side length 1. Compute the number of rectangles defined by line segments of the grid.

   (b) A triangular grid is obtained by tiling an equilateral triangle of side length 5. Compute the number of parallelograms bounded by line segments of the grid.

6. Prove the following identities using a combinatorial argument:

   (a) $\binom{n}{r}\binom{r}{k} = \binom{n}{k}\binom{n-k}{r-k}$

   (b) $\binom{n}{0}^2 + \binom{n}{1}^2 + \cdots + \binom{n}{n}^2 = \binom{2n}{n}$

   (c) $\binom{n}{0} + \binom{n+1}{1} + \cdots + \binom{n+k}{k} = \binom{n+k+1}{k}$

7. Compute:

$$\sum_{k=1}^{n} \binom{n}{k} k$$

8. Let n be a positive integer. Determine the number of lattice paths from $(0,0)$ to (n, n) using only unit up and right steps, such that the path stays in the region $x \geq y$.

9. Compute the number of permutations $x_1, \ldots, x_6$ of the integers $1, \ldots, 6$ such that $x_{i+1} \leq 2x_i$ for all i, $1 \leq i < 6$.

10. A collection of 8 cubes consists of one cube with edge-length k for each integer k, $1 \leq k \leq 8$. A tower is to be built using all 8 cubes according to the rules:

 • Any cube may be the bottom cube in the tower.

 • The cube immediately on top of a cube with edge-length k must have edge-length at most $k + 2$.

 Let T be the number of different towers than can be constructed. What is the remainder when T is divided by 1000?

11. Call a set of integers spacy if it contains no more than one out of any three consecutive integers. How many subsets of $\{1,2, \ldots, n\}$ including the empty set, are spacy?

12. A fair coin is to be tossed 10 times. Compute the probability that heads never occur on consecutive tosses.

13. Let $d_1, d_2, \ldots, d_{12}$ be real numbers in the open interval $(1, 12)$. Show there exists distinct indices i, j, k such that d_i, d_j, d_k are side lengths of an acute triangle.

14. Let p be the probability that, in the process of repeatedly flipping a fair coin, one will encounter a run of five heads before one encounters a run of two tails. Compute p.

15. Let A, B, C and D be the vertices of a regular tetrahedron each of whose edges measures 1 meter. A bug, starting from vertex A, observes the following rule: at each vertex it chooses one of the three edges meeting at that vertex, each edge being equally likely to be chosen, and crawls along that edge to the vertex at its opposite end. Let p be the probability that the bug is at vertex A when it has crawled exactly 7 meters. Find the value of p.

16. Let S be the set of all n −digit numbers created using the digits $\{1,2,3,4\}$. How many elements of S are multiples of 3?