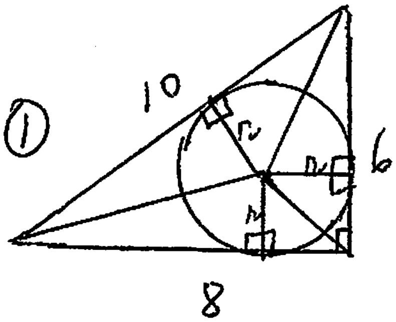
Solutions

(1)

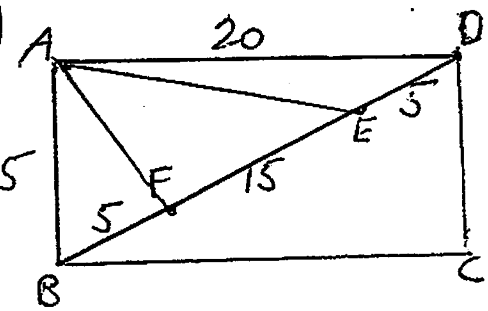


Area of right triangle $= \frac{1}{2} \times 6 \times 8 = 24$

Area of 3 triangles $= \frac{1}{2} r 8 + \frac{1}{2} r 6 + \frac{1}{2} r 10 = \frac{1}{2}(24r) = 12r$

$12r = 24 \quad r = 2$

(2)



Area of $\Delta ABD = 150$

$FE = \frac{3}{5} BD$

Area of $\Delta AFE = \frac{3}{5} 150 = 90$

(3) Intuitively, the other two sides each have length 5. Hero's formula proves this, Let one side have length b, the other $10-b$

$A = \sqrt{8(8-6)(8-b)(8-(10-b))} =$

$4\sqrt{(8-b)(b-2)}$

this will have a maximum area when $b=5$.

Area $= 4(3) = 12$

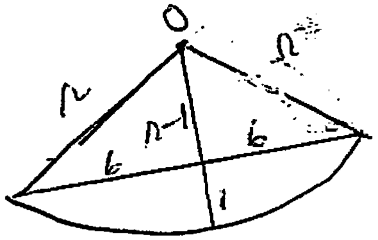
(4) Since the external angles measure $\frac{360}{x}$, they will be integral if x is a factor of 360.

$360 = 2^3 3^2 5^1$, so 360 has $4 \times 3 \times 2$ or 24 factors

We must eliminate 1 and 2. Since they do not produce polygons

$24 - 2 = 22$

(5)



$6^2 + (r-1)^2 = r^2$

$2r = 37$

$d = 37$

⑥ The volume of the pyramid is
$$V = \frac{1}{3}Ah.$$
The area of the base is 1
The height is one leg of a right triangle, the other being half the diagonal of the square, $\frac{\sqrt{2}}{2}$ and hypotenuse the edge, 1.

$h^2 + \left(\frac{\sqrt{2}}{2}\right)^2 = 1^2 \quad h = \frac{\sqrt{2}}{2}$

$V = \frac{1}{3} \cdot 1 \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{6}$

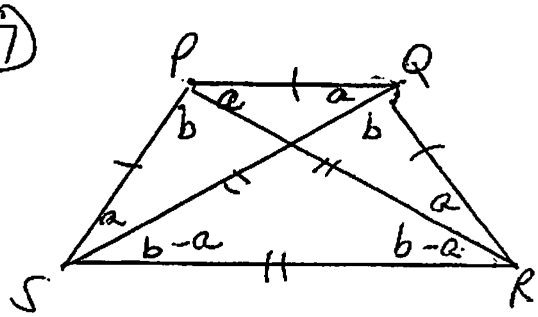
⑧ If the side of the cube is X, the diameter of the sphere, 6 is $X\sqrt{3}$. $X\sqrt{3} = 6 \quad X = \frac{6}{\sqrt{3}}$

$V = X^3 = \frac{216}{3\sqrt{3}} = \frac{216\sqrt{3}}{9} = 24\sqrt{3}$

⑩ $\frac{(N-2)180}{N} = \frac{3}{2}\frac{(m-2)180}{m}$

Solving for N algebraically,
$N = \frac{4m}{6-m}$. Since the only m's that will produce positive integers for N are 3, 4 and 5, the solutions are (3,4) (4,8) and (5,20)

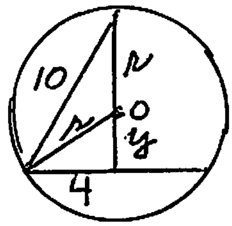
⑦



In ΔPQR, $b + 3a = 180$
In ΔPRS, $3b - a = 180$
$a = 36 \quad b = 72$
$\angle S = b = 72°$

⑨ 15 will be the hypotenuse once, 9, 12, 15.
To be a leg $c^2 - a^2 = 225$
$(c-a)(c+a) = 225$
These numbers could be 1×225, 3×75, 9×25 and 5×45
Each one will produce a Pythagorean triple. For example
$c - a = 1 \quad c + a = 225$
$c = 113 \quad a = 112$

(11)



The height of the triangle $h=\sqrt{84}$

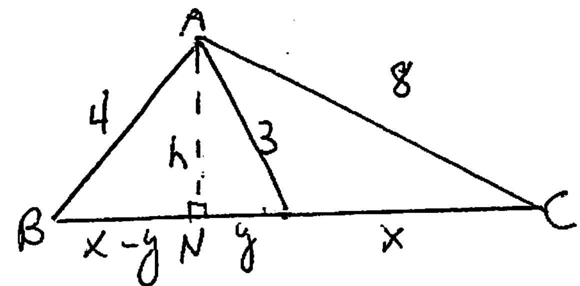
$r^2 = y^2 + 4^2 = (h-r)^2 + 4^2 =$

$h^2 - 2hr + r^2 + 16 = r^2$

$2hr = 16 + h^2$

$r = \frac{16+h^2}{2h} = \frac{(16+84)}{2\sqrt{84}} =$

$\frac{50}{\sqrt{84}} = \frac{25\sqrt{21}}{21}$



Then $h^2 + (x+y)^2 = 64$

$h^2 + (x-y)^2 = 16$

$h^2 + y^2 = 9$

Adding the first 2 and subtracting twice the second,

$2x^2 = 62 \quad x = \sqrt{31}$

$BC = 2\sqrt{31}$

(13a) $x = \sqrt{3\sqrt{24x}} \qquad x^2 = 3\sqrt{24x} \qquad x^4 = 216x \qquad x = 6$

(b) $x = \sqrt{3\sqrt[3]{288x}} \qquad x^2 = 3\sqrt[3]{288x} \qquad x^6 = 27(288x) \qquad x = 6$

(c) $x^3 = 3 \qquad x = \sqrt[3]{3}$

There was a young man from Trinity
Who found the square root of infinity
But all of the digits
Gave him the fidgets
So he chucked math and took up divinity.

O sibili, si emgo
Fortibuses i naro.
O nobili, demis trux
Vatis inem? Caus andux.