NYC Math Team Practice Session
March 18, 2016
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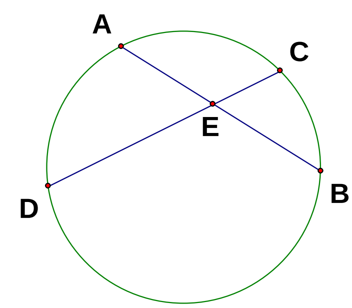
Point, What Great Power You Have!

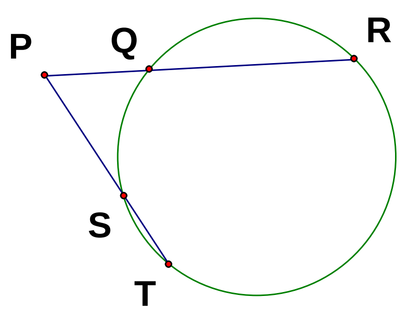
Here we are. Surely you have seen this theorem before, but perhaps you do not remember it from then or knew it with a different name. Nevertheless, this is one of the most important theorems that you will need to know if you wish to handle circle problems well! Such power, all concentrated at a single point…

1) Consider a circle with O with points R and S on the circle such that $m\overset{\frown}{RS} = 50°$. Let T and U also be points on the circle. Must it be true that $m\angle RTS = m\angle RUS$? Why or why not?

The Basic Cases:



2) In the diagram at right, chords $\overline{CD}$ and $\overline{AB}$ intersect at point E. Let $AE = 5$, $EB = 4$, and $DE = 7$. Compute the length of $\overline{DC}$. (*Hm. This problem looks very similar to that of something we have seen before…perhaps something is missing? Look for* similarity *to problems that came before.*)



3) Let's have a look outside of the circle this time. Here, $PQ = 4$, $QR = 8$, and $ST = 2$, compute PS. (*Note that there is a subtle difference between what you have here and what is in problem #2. Still, I imagine you can do the same thing and get a very similar result…*)

4) Consider what happen as T is moved along the circle towards S. How does that affect the result of problem #3? What ends up happening to our result as T and S become the same point?

The Basic Problems:

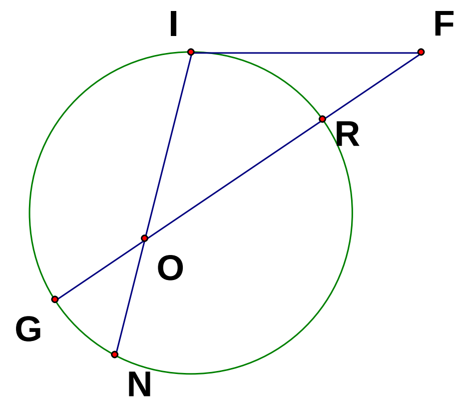
5) (1985 ARML Individual #1) In a circle, chords $\overline{AB}$ and $\overline{CD}$ intersect at R. If $\frac{AR}{RB} = \frac{1}{4}$ and $\frac{CR}{RD} = \frac{4}{9}$, compute the ratio $\frac{AB}{CD}$.

6) Let chords $\overline{GH}$ and $\overline{IJ}$ intersect at a point M within circle O such that $\overline{GH} \perp \overline{IJ}$. If $GM = 2HM = 6IM = 12$, compute GJ.

7) Suppose that Earth is a perfect sphere roughly 8000 miles in diameter. I am riding in a fighter jet 6 miles above the surface of the earth. Approximately how far away is the horizon? (*That is, let* Q *be the furthest point on the surface of the Earth that is in my view. How far away is* Q?)

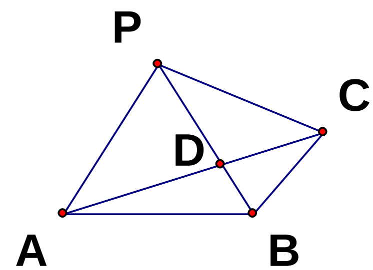
8) Secants $\overline{AB}$ and $\overline{AF}$ are drawn such that A is outside of the circle and B and F lie on the circle. $\overline{AB}$ intersects the circle at C and $\overline{AF}$ intersects the circle at D. E is a point on $\overline{AF}$ such that $AD = DE = EF$, and C is the midpoint of $\overline{AB}$. Compute the ratio $\frac{AC}{AD}$.

The Not-So-Basic Problems:

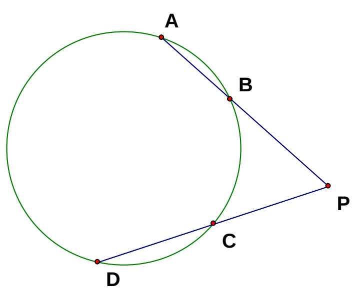


9) (3/00 Mandelbrot #7) Refer to the diagram at right, where $\overline{IF}$ is a tangent drawn to the circle, and chord $\overline{IN}$ intersects secant $\overline{FG}$ at *O*. If $IF = 21\sqrt{2}$, $IO = 20$, $ON = 12$, $RF = 18$, and $OR > GO$, compute *OR*.

10) (1995 AHSME #28) Two parallel chords in a circle have lengths of 10 and 14. These chords are situated 6 units apart. A chord is drawn midway between the two. Compute the length of that chord. (*Draw a good picture, and keep all of your measures straight. This can take a while.*)



11) (1997 AHSME #26) In the diagram at right, *P* is equidistant from *A* and *B*, $m\angle APB = 2 \times m\angle ACB$, and $\overline{AC}$ intersects $\overline{BP}$ at *D* such that $PD = 2$ and $DB = 1$. Compute $AD \times CD$. (*Now isn't this a strange set of givens…and a strange goal! There's not even a circle here! Or is there?*)



12) (1992 AHSME #27) In the diagram to the left, $BP = 8$, $AB = 10$, $CD = 7$, and $m\angle P = 60°$. Compute the area of the circle. (*Hoo boy. This is a toughie. First off, determine what segments you can. Consider—why is it important that I told you the angle's measure?*)

13) (1997 ARML Team #2) An equilateral triangle ΔABC is inscribed in circle *O*. Let *D* and *E* be the midpoints of $\overline{AC}$ and $\overline{AB}$ respectively. When extended, ray $\overrightarrow{DE}$ intersects the circle at *F*. Then, the ratio $\frac{DE}{EF}$ can be written in the form $\frac{a+\sqrt{b}}{c}$, where *a*, *b*, and *c* are integers, and the ratio is in simplest form. Compute the ordered triple (a,b,c).

14) (2013 Stanford Math Tournament Geometry Tiebreaker #1) A circle of radius 2 is inscribed in equilateral ΔABC. The altitude from *A* to $\overline{BC}$ intersects the circle at a point *D* not on $\overline{BC}$. Let $\overline{BD}$ intersect the circle at a point *E* that is distinct from *D*. Compute *BE*.

The Very-Definitely-Not-So-Basic Problems

15) (2013 HMNT Team #6) Points *A*, *B*, and *C* lie on a circle ω such that $\overline{BC}$ is a diameter. $\overline{AB}$ is extended past *B* to a point B' and $\overline{AC}$ is extended past *C* to a point C' such that $\overline{B'C'} \parallel \overline{BC}$ and tangent to the circle ω at point *D*. If $B'D = 4$ and $C'D = 6$, compute *BC*.

16) (2013 HMNT Team #7) In equilateral ΔABC, a circle ω is drawn such that it is tangent to all three sides of the triangle. A line is drawn from *A* to a point *D* on side $\overline{BC}$ such that $\overline{AD}$ intersects ω at points *E* and *F*. If $EF = 4$ and $AB = 8$, compute $|AE - FD|$.