Group A/Level A
LZ

REVIEW PROBLEMS (R1 –R6)

R1. How many distinct positive integer factors (divisors) has the number 2^{15} ? Be sure to include 1 and the number itself.

R2. List all 12 divisors of $(7^3)(13^2)$

R3. How many distinct positive integer factors has each of the following numbers ?
(i) 4^{15} (ii) $(2^{10})(3^5)$ (iii) 60^6 (iv) $(4^5)(6^4)$ (v) 5400 (vi) (25)(27) (vii) (15)(45)

R4. How many integers between 10 and 200 have an odd number of factors?

R5. Find ALL possible values of the missing digits **A** and **B**, such that the six digit number 142**A**5**B** is divisible by (a) 3 but not 9 (b) 4 (c) 8 (d) 11

R6. Find ALL possible values of the missing digits **A** and **B**, such that the four digit number 2**A**5**B** is divisible by (a) 99 (b) 999

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B1. If $(t^3 + 3t - 2)(2t^2 - t + 1) = (at^5 + bt^4 + ct^3 + dt^2 + et + f)$ is an identity, compute the sum $(a + b + c + d + e + f)$.

B2. Find the remainder when

(a) $t^3 + 4t^2 - 4t + 4$ is divided by $t - 4$.

(b) $(t + 4)^2 + (t + 3)^3 + (t + 2)^4$ is divided by $t + 1$.

(c) $t^n - 1$ is divided by $t - 1$, where n is a positive integer

(d) Generalize the result obtained in part (c)

B3. Find the value of k such that $(t - 3)$ is a factor of $(kt^3 - 6t^2 + 2kt - 12)$

B4. Find the remainder when $2t^3 + t^2 + t + 1$ is divided by (a) $t - 1$ (b) $2t - 1$.

B5. Find the remainder when $t^{40} - 1$ is divided by $t^2 - 1$.

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TAKE-OUT (M1 –M8, F1 – F5)

M1. Find ALL possible values of the missing digits **A** and **B**, such that the six digit number 872**A**5**B** is divisible by (a) 99 (b) 999 (c)

M2. Let $30! = (3^k)(N)$ where N and k are positive integers and N is NOT a multiple of 3. Compute k.

M3. Find the Highest Common Factor of (a) 124 and 132 (b) 96 and 44 (c) 10125 and 10126

M4. Find $[2 \cdot 3^5 \cdot 5^3, 2^2 \cdot 3^3 \cdot 7^2]$ and $(2 \cdot 3^5 \cdot 5^3, 2^2 \cdot 3^3 \cdot 7^2)$. Recall that [a, b] means the LCM (Least Common Multiple) of a and b while (a, b) means the HCF of a and b.

M5. Find the LCM of 2, 3, 4, 5, 6, 7, 8, 9, and 10.

M6. If 3 divides (n + 1), must 3 divide (7n + 4)? Explain.

M7. Find the smallest positive integer k such that 990 divides k!

M8. A positive integer greater than 1 that is not prime is said to be ***composite***. Observe that if N = (4)(5)(6)(7) + 7, then N is composite (why?). With this in mind, find a set of seven consecutive integers all of which are composite.

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F1. The numbers 36, 49, and 64 are three *consecutive* perfect squares. The numbers 96721, 97344, and *B* are also three consecutive perfect squares. Compute *B*.

F2. The sum $1 + 2 + 3 + \cdots + n$ is the three digit integer A A A (*which has three identical digits*). Compute n.

F3. The fraction $\frac{n}{d}$ can be reduced, but the equivalent fraction $\frac{n-98}{d-147}$ cannot be reduced. [That is, n – 98 and d – 147 are relatively prime positive integers.]

Compute the *ordered pair* of positive integers (*n*, *d*).

F4. The equation $(x + 17)(x - 17)(x - 34) = 2014$ has only one positive integer root. Compute that root.

F5. When the first (leftmost) digit of the positive integer *N* is removed, the new number formed by the remaining digits (in their original order) is *one-fifty-seventh* of *N*. What is the removed digit ? [Note: The leftmost digit may not be zero.]