NYC Math Team Practice - October 16, 2015

Polynomials Problem Set

Remainder Theorem and Division Algorithm

1.) What is the remainder when $f(x) = x^4 - 3x^3 + 7x^2 - x + 5$ is divided by $(x-3)$?

2.) What is the remainder when $f(x) = x^{10} - 2x^5 + 3$ is divided by $2x - 4$?

3.) Compute k such that when $x^3 + kx + 1$ is divided by $x + 1$, the remainder is -4.

4.) Let $P(x) = x^2 - 5px + 6p^2$. Find a prime p such that 15 is a root of $P(x) = 0$.

5.) Let $P(x) = x^3 + kx + 6$:
a.) Show that when $P(x)$ is divided by $x + 3$, the remainder is $-21 - 3k$.
b.) Compute k such that $x + 3$ is a factor of $P(x)$.

6.) When $x^2 + 2px - 3q^2$ is divided by $x - p$, the remainder is 0. Show that $p = \pm q$.

Challenge Problems:

7.) $P(x)$ is a polynomial with real coefficients. The sum of the coefficients of $P(x)$ is 4. The remainder when $P(x)$ is divided by $(x-2)$ is 9. What is the remainder when $P(x)$ is divided by $x^2 - 3x + 2$?

8.) Find the remainder when $x^{100} - 4x^{98} + 5x + 6$ is divided by $x^3 - 2x^2 - x + 2$

Viete's Formulas:

9.) If $P(x) = 3x^2 + 9x + 19$, find the sum and the product of the roots

10.) Given: $P(x) = x^2 + kx + 2k, k \neq 0$. One root of $P(x) = 0$ is twice the other. Compute the value of k AND the roots.

11.) The roots of $5x^3 + 4x^2 - 8x + 6 = 0$ are a, b, and c, where $a, b, c \in \mathbb{R}$. Compute the value of $a(1 + b + c) + b(1 + a + c) + c(1 + a + b)$.

12.) The roots of $x^2 - 26x + c = 0$ are r and s. If $19r + 94s = 1994$, compute c.

13.) (1991 NYSML I-2) Let $f(x) = x^2 + bx - 9$ and let $g(x) = x^2 + ax + c, a, b, c \in \mathbb{R}$. If the roots of $f(x) = 0$ are m and n, the roots of $g(x) = 0$ are $-m$ and $-n$, and $h(x) = f(x) + g(x)$, solve the equation $h(x) = 0$

Challenge Problems:

14.) The polynomial $P(x) = x^3 + ax^2 + bx + c = 0$ has the property that the arithmetic mean of its roots, the product of its roots and the sum of its coefficients are all equal to one another. If $P(0) = 2$, compute the value of b.

15.) For nonzero constants c and d, the equation $4x^3 - 12x^2 + cx + d = 0$ has two real roots that sum to 0. Compute $\frac{d}{c}$.

16.) The roots of $x^3 - 5x^2 + 7x - 9 = 0$ are a, b and c. The roots of $x^3 + px^2 + qx + r = 0$ are $(a + b), (a + c)$ and $(b + c)$. Compute the ordered pair (p, q).

17.) (1987 NYSML T-9) Let $P(x) = x^3 + 5x^2 + px + q$ and let $Q(x) = x^3 + 7x^2 + px + r$, where $p, q, r \in \mathbb{R}$.
If $P(x) = 0$ and $Q(x) = 0$ have two roots in common, and the third root of each polynomial is c and d respectively, then find (c, d). This means to express your answer as an **ordered pair**.