New York Math Team Practice
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Warm Up Problems

1. Given m and n are roots of $7x^2+9x+21=0$, find the value of $(m+7)(n+7)$.

2. A parabola has vertex $(4,3)$ and an axis of symmetry parallel to the y-axis. If one x-intercept is 1, find the other.

3. There are two values of a for which the equation $4x^2+ax+8x+9=0$ has only one solution for x. Find them.

   2005 AMC 10A #10

4. A parabola has a vertex at $(4,-5)$ and two x-intercepts, one positive and one negative. If the parabola is the graph of $y=ax^2+bx+c$, which of a, b and c must be positive?

   ASHME 1998 #14

5. Find the range of $H(x)=\dfrac{x}{x^2+2x+6}$.

Quadratic Problems

1. If $(1-3k)x^2+3x-4=0$ has one real root, find the largest integer value k can have.

2. A quadratic passes through $(-1,-2)$, $(1,0)$, $(2,7)$ and $(3,y)$. Find y.

   NYCIML SF79 #7

3. If $c>a>0$, and if $a-b+c=0$, find the larger root of $ax^2+bx+c=0$.

   NYCIML SF76 #27

4. Compute all real values of a such that $x^2+ax+a=-8$ has two distinct positive roots.

   NYCIML JF06 #18

5. Find the number of quadratic equations $f(x)=ax^2+bx+c$ with integer coefficients that contain the points $(0,0)$ and $(15,225)$.

   ARML 2013 I7

6. Given that $x^2+y^2=14x+6y+6$, what is the largest possible value that $3x+4y$ can have?

   ASHME 1996 #25

7. The circle $(x-5)^2+(y-3)^2=25$ intersects the x-axis at A and B. Find the equations of all parabolas, with vertical axis of symmetry, whose only points in common with the circle are A and B.

8. Given the circle centered at O with points of tangency at D, M and N, if $m\angle ABC=60°$ and if $AB=1$, find the radius of the circle.

   ASHME 1997 #19

9. Minimize $\frac{x^2-2x-1}{(x-4)^2}$.

10. If $ax^2+bx+c=0$ has real root *r*, and $-ax^2+bx+c=0$ as real root *t*, with $a,c\neq 0$, prove that a real root *q* exists to $\frac{a}{2}x^2+bx+c=0$ such that *q* is between *r* and *t*.

11. Find all integers *n* such that $n^2+13n+3$ is a perfect square.

12. Consider the following quadratic equation:

$$H(x)=(ax-p)^2+(bx-q)^2+(cx-r)^2.$$

a. If all coefficients are real, explain why $H(x)\geq 0$ for all real *x*.

b. Under what condition will $H(x)=0$?

c. Use the discriminant to show that $(ap+bq+cr)^2\leq(a^2+b^2+c^2)(p^2+q^2+r^2)$.
This is the **Cauchy-Schwarz Inequality** for the case of three pairs of numbers.

d. Can you generalize this to show that for real numbers $a_1,a_2,a_3\ldots a_n$ and $b_1,b_2,b_3\ldots b_n$, the quadratic $H(x)=(a_1x-b_1)^2+(a_2x-b_2)^2+\cdots+(a_nx-b_n)^2$ leads to

$$\left(\sum_{i=1}^{n}a_ib_i\right)^2\leq\left(\sum_{i=1}^{n}a_i^2\right)\left(\sum_{i=1}^{n}b_i^2\right)?$$