NYC Math Team Practice Session — Stan Kats
Number Theory — 9/25/15

1. Compute all ordered pairs of relatively prime positive integers (x, y) such that xy is equal to:

(a) 16

(b) 36

(c) 225

2. Use your results from the previous question to make a conjecture and prove the conjecture.

3. A Pythagorean Right Triangle (PRT) is a right triangle whose sides all have positive integer lengths. Find all PRTs with a leg of:

(a) 7

(b) 8

(c) 9

4. Show that if (x, y, z) is a Primitive Pythagorean Triple (PPT) then:

(a) exactly one of x, y, z is divisible by 4

(b) exactly one of x, y, z is divisible by 5

(c) either x or y is divisible by 3 (assume $x, y < z$)

5. Find all PRTs (whenever possible) having an hypotenuse of:

(a)17

(b)25

(c)31

(d)65

(e)58

6. Find all P(rimitive)PRTs whose areas are (numerically) twice their perimeters.

7. Let $d_1, d_2, \dots, d_k$ be the divisors of the positive integer n, where:

$$1 = d_1 < d_2 < d_3 < \cdots < d_k = n$$

If $(d_7)^2 + (d_{15})^2 = (d_{16})^2$, compute all possible values for d_{17}.

8. Show that there are no PRTs such that both legs are perfect squares.