New York City Math Team Practice Session Sept. 25, 2015
Group A/Level A
LZ

*In the following exercises, all variables represent positive integers unless noted otherwise*

1. (a) Is the number 269 prime or composite? (b) Is the number 403 prime or composite?

2. (a) If 3 is a factor of $(39 + N)$, must 3 be a factor of N?
   (b) If 3 is a factor of $(35 + N)$, must 3 be a factor of N?
   (c) If 3 is a factor of $(a + b)$, must 3 be a factor of a *and* 3 be a factor of b?
   (d) If 3 is a factor of a and 3 is a factor of b, must 3 be a factor of $13a + 17b$?
   (e) If 3 is a factor of N and 4 is a factor of N, must 12 be a factor of N
   (f) If 6 is a factor of N and 4 is a factor of N, must 24 be a factor of N?
   (g) If 3 is a factor of 15N, must 3 be a factor of N?
   (h) If 3 is a factor of 17N, must 3 be a factor of N? How does this differ from (g)?
   (i) If 6 is a factor of the product (x)(y), where x and y are integers, must 6 be a factor of either x or y?
   (j) If 3 is a factor of the product (x)(y), where x and y are integers, must 3 be a factor of either x or y?

3. Find all of the prime factors of (a) $10^5$ (b) 10!

4. Recall that the notation 15! means $15\times14\times13\times \cdots \times3\times2\times1$.
   If 15! = 130767**A**368000, find the missing digit **A**.

5. Determine the missing digit, **A**, in the product (9966334)(9966332) = 99327**A**93466888

6. Find *all* possible values of the digit **A** such that the ten digit number 275453165**A** is divisible by (a) 3 (b) 9 (c) 6 (d) 4 (e) 8 (f) 11 (g) 7 (h) 13

7. Find ALL possible values of the missing digits **A** and **B**, such that the nine digit number 275453**A**6**B** is divisible by (a) 3 (b) 9 (c) 6 (d) 11 OPTIONAL(e) 7

8. The six digit number 739**ABC** is divisible by 7, 8, and 9. Find all possible values of the three digit number **ABC.**

9. Show that the sum of any nine consecutive integers is divisible by 9. Generalize (Be careful!!)

10. Notice, for example, that $276^2 = 76176$. Show that if the integer N terminates in 76, then the integer $N^2$ also terminates in 76. Is the converse true?

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11a. How many distinct positive integer factors (divisors) has the number $2^{15}$ ? Be sure to include 1 and the number itself.

11b. List all 12 divisors of $(7^3)(13^2)$

12. How many distinct positive integer factors has each of the following numbers ?
(i) $4^{15}$ (ii) $(2^{10})(3^5)$ (iii) $60^6$ (iv) $(4^5)(6^4)$ (v) 5400 (vi) (25)(27) (vii) (15)(45)

13a. Find an integer, greater than 1,000,000 that has exactly 15 distinct positive integer factors.

13b. Find five different integers, each of which has exactly 12 distinct positive integer factors.

13c. How many integers between 10 and 200 have an odd number of factors?

13d. For the number $(2^{10})(3^5)$, compute the number of (a) even factors (b) the number of odd factors (c) the number of perfect square factors (d) the number of perfect cube factors.

13e. What is the smallest positive integer having precisely (a) 13 divisors ? (b) 12 divisors?

14. (a) Can a perfect square have a units digit of 3? Explain.
(b) Show that the square of every odd positive integer is 1 more than a multiple of 8.

15. Show that $x^2 - y^2 = 74$ has no solutions in integers

16. The numerical sum of the area and the perimeter of a rectangle is 44. Find all possible dimensions of the rectangle if the length and width are integers.

17. Show that the smallest square that is expressible as the sum of three consecutive positive integers is 9. Find the next two such squares.

LZ 9/25/15

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