Didya take another look at digits?

Name:_____   Date:_____

Try this problem. How do you represent N algebraically?

1. (1984 NYCIML S7) The number N has three digits when written in base ten notation. Its cube root is the sum of its three digits. Find N.

Make sure you are able to represent numbers algebraically, in terms of their digits. By doing so, you are giving yourself information to work with.

2. (1982 NYCIML F3) If the hundreds digit of a three-digit number is increased by n, and both the tens and units digits are decreased by n, the new number formed is n times greater than the original number. Find the original number.

3. (1990 ARML Team 1) In the addition problem at the right, each letter represents a different digit. Compute the difference between the greatest possible value for this sum and the greatest possible odd value for this sum.

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   AB
   CD
   EF
   GH
+  IJ
――――
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4. (1975 NYCIML S25) If A and B are digits and the base ten numeral $30AB5$ can be expressed as the product $225n$, find all positive integral values of n.

In the previous problem, the divisibility properties of an unknown number can inform you about its digits. Write down the divisibility test for 9, so that you can use it in the next problem. Can you prove the divisibility test for 9?

5. (1990 NYSML Individual 1) When multiplied out, 15! is equal to $130767A368000$. Compute the missing digit A.

6. (1983 NYCIML F10) The sequence $\{a_i\}$ is defined as follows: $a_1 = 3^{1983}$, and for $i > 1$, a_i is the sum of the digits in the decimal representation of a_{i-1}. Find the numerical value of a_{10}.

7. (1979 NYCIML F2) The number N, represented by the decimal numeral ABC, is divided by the decimal numeral AC. The quotient is 9 and the remainder is 0. What is the maximum value possible for N?

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8. (1994 ARML Team 5) In the addition below, each letter represents a different digit. Compute the digit that J represents.

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   ABC
   DEF
+  GHI
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  1J32
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9. (1982 NYCIML F28) How many four-digit numbers, made up only of the digits 1, 2, 3, are multiples of 9?

If you haven't done so, go back and figure out the proof of the divisibility test for 9. It will give you some insights for problems without multiples of 9.

10. (1994 ARML Tiebreaker 2) Compute the missing digit, A, in the product

$$(9966334) \cdot (9966332) = 99327A93466888.$$

11. (2014 SMT General Tiebreaker 3) Let n be the number which consists of the first 2014 positive integers concatenated together. Let $f(x)$ be the sum of the digits of x, and let $g(x)$ be the value obtained by applying f repeatedly to x until it converges to a single value. Compute $g(n)$.

12. (2014 SMO Junior 35) The number 2^{29} has exactly 9 distinct digits. Which digit is missing?

A divisibility test for 11 also exists; it's very similar to the divisibility test for 9. Find it and prove it.

13. (1980 NYCIML S9) Find the largest three-digit number (written in decimal notation) that is divisible by 22 and such that the sum of the units digit and the tens digit is 11.

14. (2012 SMT Advanced Topics 1) Define a number to be *boring* if all the digits of the number are the same. How many positive integers less than 1000 are both prime and boring?

These are some miscellaneous problems related to the digits of a number. They are not completely disconnected, though; can you find the connection?

15. (1983 NYCIML S17) If all numbers are written in base ten notation, how many digits are in the numeral representing 61224^2?

16. (1982 NYCIML S1) The units digit of the difference of the squares of two integers (written in base ten notation) is 2. What is the units digit of the sum of these two squares?

17. (2004 AMC 12B 25) Given that 2^{2004} is a 604-digit number whose first digit is 1, how many elements of the set $S = \{2^0, 2^1, 2^2, \ldots, 2^{2003}\}$ have a first digit of 4?