I8. Consider the given identity $(x-6)\cdot P(x) = x\cdot P(x-1)$. If we set $x=0$, we find that $P(x)=0$, so 0 is a root of $P(x)$. If we set $x=1$, we find that the $(1-6)\cdot P(1) = 1\cdot P(0) = 0$, so 1 is another root. Similarly, we see that 2, 3, 4, and 5 are roots. But when we set $x=6$, both sides of the identity are 0, so it doesn't tell us anything.

Since 0 through 5 are roots, we know that $P(x) = x(x-1)(x-2)(x-3)(x-4)(x-5)Q(x)$. Plugging this into the identity, we see $Q(x) = Q(x-1)$ for all x.

It would be reasonable to guess that $Q(x)$ must be a constant, and since $P(x)$ is monic, this constant must be 1. So it remains to plug in -1 into $P(x) = x(x-1)(x-2)(x-3)(x-4)(x-5)$ to get $(-1)^6\cdot 6! = 720$.

[Without guessing, we can prove that if $Q(x) = Q(x-1)$ for all x, $Q(x)$ must be constant. Consider $T(x) = Q(x) - Q(0)$. That's a polynomial, with $T(0) = 0$ and $T(x) = T(x-1)$ for all x. This means that $T(n) = T(n-1) = T(n-2) = ... = T(0) = 0$ for all positive integers n. But a polynomial can't have infinitely many roots unless it is identically zero. So it must be that $T(x) = 0$. But that means that $Q(x) = Q(0)$ for all x, i.e. it is indeed constant.]