NYCMT Team Questions - practice 02-06-15

T1. Find the greatest possible area of a rectangle whose perimeter is 50.

T2. Find the remainder when 1•3•5•...•2009 + 2•4•6•...•2010 is divided by 2011.

T3. The largest prime factor of $1994^4 + 4$ has four digits. Compute the second largest prime factor.

T4. Compute the number of integer solutions (x, y) to

$$xy - 18x - 35y = 1890.$$

T5. In a square ABCD with side length 1, rays from A are drawn such that vertex C lies between them. From B and D, perpendiculars are dropped to each of the two rays. If the angle between the two rays is 60°, what's the maximum area of the quadrilateral whose vertices are the feet of these perpendiculars?

T6. Prove that $a^2+b^2+c^2 \geq ab+ac+bc$, where a, b, c are real.

T7. Prove that for any integer $n > 1$, $n! < \left(\frac{n+1}{2}\right)^n$.

T8. Prove that for nonnegative x, y, z , if $xyz \geq 1$ then

$$(x+1)(y+1)(z+1) \geq 8 .$$

T9. Prove that if a, b, c are positive, then $(a+b)(b+c)(a+c) \geq 8abc$.