TEAM PROBLEM SOLUTIONS FOR 2013

T1. Draw vertical and horizontal grid lines to enclose ΔABC in a rectangle. Subtracting the areas of the three "outer triangles" from the rectangle, we get Area of ABC = 14, so the Area of parallelogram ABCD = **28**. Alternatively, use determinants to get the area of triangle ABC.

T2. Let the height and length of the picture be a and b respectively, and the corresponding outer dimensions of the border be a+2 and b+2. ab = (a+2)(b+2) - ab $\Rightarrow$ 2a+2b-ab+4 = 0 $\Rightarrow$ (a-2)(b-2) - 8 = 0 $\Rightarrow$ (a-2)(b-2) = 8. The two possible factorings of 8 are 1•8 and 2•4, leading to ab = 3•10 or 4•6. The larger (area) picture is **3 by 10**.

T3. A is either in quadrant III or IV. Let the corresponding reference angles each be x. A = 180+x or A = 360-x, so their sum is **540°** .

T4. Here's three methods, each based on $\Delta GAH \sim \Delta CBH$, so AH:HB = GH:HC = $\sqrt{3} : \sqrt{27}$ = 1:3 . K is for area.

{1} $K_{GAH} : K_{GDC} = 1^2 : 4^2$ = 1:16, so K_{GDC} = 16(3) = 48. Thus K_{AHCD} = 48-3 = 45, and K_{ABCD} = 45+27 = **72**.

{2} Draw AC. Since ΔACH and ΔBCH have the same altitude (from C), their areas are in the same ratio as their bases. Thus $K_{ACH} : K_{BCH}$ = 1:3 = 9:27, so K_{ACH} = 9. Thus $K_{ABCD} = 2(K_{ACB})$ = 2(9+27) = **72**.

{3} Choose J on DC such that HJ || AD. $K_{HJC} = K_{HBC}$ [since HC is a diagonal of parallelogram HJCB] = 27, so K_{HJCB} = 54. Also, since parallelograms ADJH and HJCB have the same height, their areas are in the same ratio as their bases; therefore $K_{ADJH} : K_{HJCB}$ = 1:3 = 18:54, so K_{ADJH} = 18. Then K_{ABCD} = 18+54 = **72**.

T5. This sequence is n, 10n+7, 10^2n+77, 10^3n+777, ... ≡ $\underline{n}$, $\underline{n}\ \underline{7}$, $\underline{n}\ \underline{7}\ \underline{7}$, $\underline{n}\ \underline{7}\ \underline{7}\ \underline{7}$, ... The 50th term is $\underline{n}$ followed by 49 sevens. The sum of these digits is n+3+4+3. To be a multiple of 9, we need n+10 = 18, so n = **8**.

T6. Here are two methods, each using AD = DE = EB = 2x, AC = b, BC = a, AB = c.

{1} Let M be the midpoint of AB; draw CM. In a right triangle, the median to the hypotenuse is ½ the hypotenuse, so CM = c/2 = 3x. In isosceles ΔCDE, median CM will also be an altitude, so $9x^2 + x^2 = 25$, leading to 6x = $\sqrt{90}$. N = **90**.

{2} Draw line through A parallel to CB. Extend CD to meet that line at F. From similar triangles AFD and BCD, since AD is half of DB, AF = a/2 and DF = 5/2 [so CF = 15/2]. Applying the Pythagorean Theorem to AFC, $(a/2)^2 + b^2 = (15/2)^2$, leading to $\underline{a^2 + 4b^2 = 225}$. By a similar argument, we get $\underline{b^2 + 4a^2 = 225}$. Adding these two equations, $5(a^2 + b^2) = 450 = 5c^2$, so c = $\sqrt{90}$, and N = **90**.

T7. Letter the circles from top to bottom, left to right, as A; B,C,D; E,F,G. Note that their sum is 28. If the "magic sum" is S, then (A+B+E) + (A+C+F) + (A+D+G) = 3S, so $\underline{2A+28 = 3S}$. Also, (B+C+D) + (E+F+G) = 2S, so $\underline{28-A = 2S}$. These equations lead to A=4, so E cannot be a **4**. [Any other number is possible for E, since we can interchange any two of the "vertical" lines, or interchange the horizontal lines.]

T8. Since $(½)ah_a = (½)bh_b = (½)ch_c$, and $h_a:h_b:h_c$ = 6:4:3, we get a:b:c = 2:3:4 . Let the sides of the Δ be a=2x, b=3x, c=4x. Then by Law of Cosines, cos C = $(4x^2 + 9x^2 - 16x^2)/2(2x)(3x)$ = -3/12 = **-1/4** .

T9. Call the horizontal chord EF and the vertical chord GH. Draw a chord || and = to EF, but above O; draw a chord || and = to GH, but to the right of O. These four chords divide the circle into 9 sections. The "corner four" each = S_1; the "outer two" of the middle horizontal set each = $S_2 - S_1$; the "top and bottom two" of the middle vertical set each = $S_4 - S_1$. Now we have $S_3 = (S_4 - S_1) + (S_1) + (S_2 - S_1) + 24$, so $(S_1 + S_3) - (S_2 + S_4)$ = **24**.

T10. Let AB = d and Winnie's usual rate be r mph. Her usual trip takes d/r mph. Expressing minutes in hours, $\frac{d}{r} - \frac{d}{4} = \frac{15/2}{60}$ and $\frac{d}{7/2} - \frac{d}{r} = \frac{15/4}{60}$. Dividing these equations [the d's will cancel] leads to $\frac{28-7r}{8r-28} = 2$, so r = F/G = **84/23** . [d will be 5¼ miles. The numbers 7½ and 3¾ may be a hint that division is helpful.]